(a) The following is a pattern for some legal strings in a language:

\[ a \in A \{0,1\} \quad b \in B \{0,1\} \quad c \in C \{1,n\} \quad d \in D \{1,1\} \]

where \( A \) is a finite set of characters from the alphabet, \( \Sigma \); similarly for \( B, C, D \). The sets \( A, B, C \) and \( D \) are disjoint. \( \{x, y\} \) indicates that the previous bracket must match at least \( x \) times but no more than \( y \) times.

(i) Specify a Deterministic Finite Automaton, \( M_1 \), that can recognise these strings only. [4 marks]

(ii) Design a Regular Grammar, \( G_1 \), which generates \( L(M_1) \). [4 marks]

(iii) Describe a set of strings in a natural language that could be generated by \( G_1 \) given an appropriate \( \Sigma \) and its subsets \( A, B, C \) and \( D \). [1 mark]

(b) We can hypothesise that matches of the following pattern are always valid constructions in English:

\[ \text{[The Noun]} \{n,n\} \quad \text{[Verb]} \{n,n\} \]

where \( \text{Noun} \) represents the coordinated members of a finite set; similarly for \( \text{Verb} \).

(i) Now consider the following English sentence which matches the pattern when \( n = 1 \):

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Provide example sentences that extend this sentence for the case when \( n = 2 \) and \( n = 3 \). [2 marks]

(ii) Assuming that these constructions are part of the English language, would this mean that English is a Context-Free Language? Justify your answer. [3 marks]

(iii) Design a grammar in Chomsky Normal Form, \( G_2 \), which generates the finite matches of the pattern. [3 marks]

(iv) Specify a Push Down Automaton, \( M_2 \), that recognises \( L(G_2) \). [3 marks]