9 Logic and Proof (lp15)

(*a*) Suppose we have the propositional symbols P_1, P_2, $\ldots$, P_n, where $n > 2$, and consider the set of clauses

$$\{\neg P_1, P_2\} \quad \{\neg P_2, P_3\} \quad \cdots \quad \{\neg P_n, P_1\}.$$

(*i*) List the satisfying interpretations (if any) of this set of clauses, with brief justification. [3 marks]

(*ii*) Regarding the set of clauses above as a single propositional formula, and using the variable ordering P_1, P_2, $\ldots$, P_n, sketch the corresponding BDD. Does the choice of variable ordering here significantly affect the size of the resulting BDD? [5 marks]

(*iii*) Briefly describe the set of clauses that would be generated by a resolution theorem prover, starting with the set of clauses above. [3 marks]

(*b*) For the following set of clauses, either exhibit a model, or show that none exists using resolution. Below, a and b are constants, while y and z are variables.

$$\{P(a, f(a)),\ P(a, b)\}$$
$$\{P(a, f(a)),\ \neg P(z, b),\ P(z, f(a))\}$$
$$\{\neg P(a, f(a)),\ P(g(y), y),\ \neg P(a, y)\}$$
$$\{\neg P(a, f(a)),\ \neg P(g(y), f(a)),\ \neg P(a, y)\}$$

[9 marks]