

3 Complexity Theory (mpf23)

(a) Define the set of Boolean expressions 2CNF and the language 2SAT over them. [2 marks]

(b) For a Boolean expression  $\phi$  in 2CNF, let  $G(\phi)$  be the directed graph with *vertices* the variables of  $\phi$  and their negation, and with *edges*  $(a, b)$  if, and only if, there is a clause  $(\neg a \vee b)$  or  $(b \vee \neg a)$  in  $\phi$ . Note that an edge  $(a, b)$  is in  $G(\phi)$  if, and only if, so is the edge  $(\neg b, \neg a)$ .

Prove that a Boolean expression  $\phi$  in 2CNF is unsatisfiable if, and only if, there is a variable  $x$  in  $\phi$  such that there are paths from  $x$  to  $\neg x$  and from  $\neg x$  to  $x$  in  $G(\phi)$ . [*Hint*: Recall that the proposition  $(\neg P \vee Q)$  is equivalently the implication  $(P \rightarrow Q)$ .] [12 marks]

(c) Argue as to whether or not 2SAT is in NL, in P, and in NP. Your answer may use the fact that NL is closed under complementation. [6 marks]