3 Complexity Theory (mpf23)

(a) Define the set of Boolean expressions $2CNF$ and the language $2SAT$ over them. [2 marks]

(b) For a Boolean expression $\phi$ in $2CNF$, let $G(\phi)$ be the directed graph with vertices the variables of $\phi$ and their negation, and with edges $(a, b)$ if, and only if, there is a clause $(\neg a \lor b)$ or $(b \lor \neg a)$ in $\phi$. Note that an edge $(a, b)$ is in $G(\phi)$ if, and only if, so is the edge $(\neg b, \neg a)$.

Prove that a Boolean expression $\phi$ in $2CNF$ is unsatisfiable if, and only if, there is a variable $x$ in $\phi$ such that there are paths from $x$ to $\neg x$ and from $\neg x$ to $x$ in $G(\phi)$. [Hint: Recall that the proposition $(\neg P \lor Q)$ is equivalently the implication $(P \rightarrow Q)$.] [12 marks]

(c) Argue as to whether or not $2SAT$ is in NL, in P, and in NP. Your answer may use the fact that NL is closed under complementation. [6 marks]