3 Complexity Theory (mpf23)

(*a*) Define the set of Boolean expressions 2CNF and the language 2SAT over them. [2 marks]

(*b*) For a Boolean expression ϕ in 2CNF, let $G(\phi)$ be the directed graph with *vertices* the variables of ϕ and their negation, and with *edges* (a, b) if, and only if, there is a clause $(\neg a \vee b)$ or $(b \vee \neg a)$ in ϕ. Note that an edge (a, b) is in $G(\phi)$ if, and only if, so is the edge $(\neg b, \neg a)$.

Prove that a Boolean expression ϕ in 2CNF is unsatisfiable if, and only if, there is a variable x in ϕ such that there are paths from x to $\neg x$ and from $\neg x$ to x in $G(\phi)$. [*Hint:* Recall that the proposition $(\neg P \vee Q)$ is equivalently the implication $(P \to Q)$.] [12 marks]

(*c*) Argue as to whether or not 2SAT is in NL, in P, and in NP. Your answer may use the fact that NL is closed under complementation. [6 marks]