2 Artificial Intelligence (sbh11)

A Boolean satisfiability problem has four variables, $x_1$, $x_2$, $x_3$ and $x_4$. A literal $l$ can be a variable or its negation, denoted $\neg l$. The formula of interest, in conjunctive normal form (CNF), is

$$f = (x_2 \lor \neg x_3) \land (\neg x_2 \lor x_3) \land (x_1 \lor x_2 \lor \neg x_4). \quad (1)$$

The aim is to find assignments to the variables such that $f$ is true under the usual rules for Boolean operations. This question addresses the use of more general constraint satisfaction to solve this problem.

(a) Give a general description of a constraint satisfaction problem (CSP).

(b) Explain how a Boolean satisfiability problem in CNF form and with $n$ variables can be converted to a CSP, also having $n$ variables and having a suitable constraint for each clause. Illustrate your answer using the 4-variable formula $f$ in (1).

(c) Explain, again using a constraint corresponding to a clause from (1), how general constraints can be converted to binary constraints. Provide a graph illustrating the problem from (1) after it has been converted to a CSP with only binary constraints.

(d) Explain, how forward checking works in the context of a general CSP. How does this benefit a CSP solver?

(e) Using the CSP equivalent you developed for (1), with only binary constraints, demonstrate how forward checking works using the sequence of assignments $x_1 = F, x_2 = F, x_4 = T$.

(f) How would you expect the solution obtained when applying forward checking to be affected if constraints were allowed to propagate more widely?