9 Semantics of Programming Languages (nk480)

(a) Suppose that \( l : \text{intref} \in \Gamma \). Now, consider the following program equivalence for \( L_1 \):

\[
(\text{if } ! l \leq 0 \text{ then } e_1 \text{ else } e_2); e_3 \equiv_{\Gamma}^{\text{unit}} (\text{if } ! l \leq 0 \text{ then } e_1; e_3 \text{ else } e_2; e_3)
\]

(i) Explain informally but carefully why this equivalence holds. [3 marks]

(ii) Using the definition of semantic equivalence, prove that this equivalence holds. [7 marks]

(b) Now, consider the following non-equivalence:

\[
e_3; (\text{if } ! l \leq 0 \text{ then } e_1 \text{ else } e_2) \not\equiv_{\Gamma}^{\text{unit}} (\text{if } ! l \leq 0 \text{ then } e_3; e_1 \text{ else } e_3; e_2)
\]

(i) Give a well-typed example exhibiting a counterexample of this equivalence. [5 marks]

(ii) Give a statically decidable condition under which the transformation is valid. [5 marks]