

9 Discrete Mathematics (mpf23)

- (a) Prove that $4^n + 6n - 1 \equiv 0 \pmod{9}$ for all natural numbers n . [5 marks]
- (b) (i) State one of the standard characterizations of the reflexive-transitive closure $R^* \subseteq A \times A$ of a binary relation R on a set A . [1 mark]
- (ii) For the given characterization for a binary relation R on a set A , prove that $R^* = \bigcup_{n \in \mathbb{N}} R_n$ where $R_0 = \emptyset$ and, for $n \in \mathbb{N}$, $R_{n+1} = \text{id}_A \cup (R \circ R_n)$. You may use standard results provided that you state them clearly. [8 marks]
- (c) Let $\mathcal{F} \subseteq \mathcal{P}(\mathbb{N})$ be a family of pairwise-disjoint subsets of natural numbers; that is, such that $\forall S, T \in \mathcal{F}. S \neq T \implies S \cap T = \emptyset$.

State whether or not the set \mathcal{F} may be uncountable and prove your claim.

[6 marks]