

COMPUTER SCIENCE TRIPOS Part IA – 2021 – Paper 2

7 Discrete Mathematics (mpf23)

(a) Without using the Fundamental Theorem of Arithmetic, prove that

$$\gcd(c, ab) = 1 \iff (\gcd(c, a) = 1 \wedge \gcd(c, b) = 1)$$

for all positive integers  $a, b, c$ . [5 marks]

(b) Let  $P(n)$  be a statement for  $n$  ranging over the set of positive integers  $\mathbb{N}^+$ .

(i) Prove that if

$$\forall m \in \mathbb{N}^+. P(m+1) \implies P(m)$$

then

$$\forall n \in \mathbb{N}^+. P(n+1) \implies (\forall k \in \mathbb{N}^+. k \leq n+1 \implies P(k))$$

[3 marks]

(ii) Prove that if

$$P(2) \wedge (\forall m \in \mathbb{N}^+. P(m) \implies P(2m)) \wedge (\forall m \in \mathbb{N}^+. P(m+1) \implies P(m))$$

then

$$\forall n \in \mathbb{N}^+. P(n)$$

[3 marks]

(c) Let  $I = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ .

In each case below define a function from  $I$  to  $I$  that satisfies the stated properties. Your answer should justify that the criteria are met.

(i) Injective but not bijective. [3 marks]

(ii) Surjective but not bijective. [3 marks]

(iii) Bijective but not the identity. [3 marks]