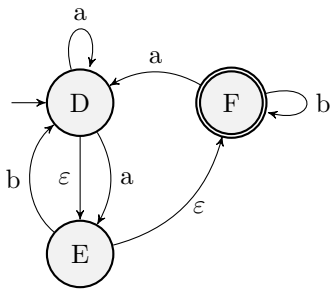
COMPUTER SCIENCE TRIPOS Part IA – 2021 – Paper 2

10 Discrete Mathematics (fms27)

(*a*) Consider the following NFA$^{\varepsilon}$, whose input alphabet is {a, b, c}.



(*i*) For each of the two strings abc and bba, state whether the automaton accepts it, with justification. [2 marks]

(*ii*) Using the subset construction, produce the full unoptimized state transition table of an equivalent DFA, *listing its states in lexicographic order* (important!) and indicating the starting and accepting states. [6 marks]

(*iii*) Give a regular expression, no longer than six symbols (metacharacters included), that describes the strings accepted by the automaton, together with an intuitive explanation for it. [*Hint:* Part (*a*)(*ii*) helps.] [4 marks]

(*b*) Consider language L_1 of strings over alphabet $\{0, 1\}$, defined inductively as follows.

$$\frac{}{\mathtt{00}}\ (0) \qquad \frac{w}{\mathtt{1}w}\ (1) \qquad \frac{w}{w\mathtt{1}}\ (2)$$

(*i*) Draw the diagram of a DFA that recognizes L_1 in no more than four states. [4 marks]

(*ii*) Considering the words in L_1 as unsigned binary numerals, let language L_2 of strings over $\{0, 1\}$ be the set of all and only the binary numerals obtained by adding 1 to any numeral in L_1 and removing any leading zeros. NB: "adding" here means arithmetic addition, not string concatenation. Produce a regular expression no longer than 11 symbols that recognizes L_2, with a clear and convincing explanation of how you derived it. [4 marks]