10 Discrete Mathematics (fms27)

(a) Consider the following NFA$^*$, whose input alphabet is \{a, b, c\}.

(i) For each of the two strings abc and bba, state whether the automaton accepts it, with justification. [2 marks]

(ii) Using the subset construction, produce the full unoptimized state transition table of an equivalent DFA, listing its states in lexicographic order (important!) and indicating the starting and accepting states. [6 marks]

(iii) Give a regular expression, no longer than six symbols (metacharacters included), that describes the strings accepted by the automaton, together with an intuitive explanation for it. [Hint: Part (a)(ii) helps.] [4 marks]

(b) Consider language $L_1$ of strings over alphabet \{0, 1\}, defined inductively as follows.

\[
\begin{align*}
00 \quad (0) \\
1w \quad (1) \\
w1 \quad (2)
\end{align*}
\]

(i) Draw the diagram of a DFA that recognizes $L_1$ in no more than four states. [4 marks]

(ii) Considering the words in $L_1$ as unsigned binary numerals, let language $L_2$ of strings over \{0, 1\} be the set of all and only the binary numerals obtained by adding 1 to any numeral in $L_1$ and removing any leading zeros. NB: “adding” here means arithmetic addition, not string concatenation. Produce a regular expression no longer than 11 symbols that recognizes $L_2$, with a clear and convincing explanation of how you derived it. [4 marks]