6 Introduction to Probability (mj201)

(*a*) A korfball player is practicing shots and has a 90% chance of scoring. Assume that their shots are independent of one another.

(*i*) Let S be the number of successful shots made in 200 attempts. Specify a suitable distribution for S including its parameters, and compute the expected value and variance. What is the probability mass function of S? [3 marks]

(*ii*) Following the experiment in Part (*a*)(*i*), let M be the number of shots made before the first miss. Specify a suitable distribution for M including its parameters, and compute the expected value and variance. What is the probability of $M > 100$? [4 marks]

(*iii*) Use a suitable distribution to approximate the probability that there are at most 3 misses in the first 200 shots. Note: you do not need to compute the final numerical value. [3 marks]

(*b*) Consider an urn containing balls labelled $0, 1, 2, \ldots, n-1$ and the experiment of drawing n of these balls uniformly and without replacement. Let X_i denote the label of the ball drawn in the i-th step, $1 \leq i \leq n$.

(*i*) For any $1 \leq i \leq n$, what is $\mathbf{E}[X_i]$ and $\mathbf{V}[X_i]$? Justify your answer. [2 marks]

(*ii*) Compute $\mathbf{Cov}[X_1, X_2]$. [4 marks]

(*iii*) Suppose now that n is an unknown parameter and you observe the absolute difference between the labels of the first two balls, that is, $Z := |X_1 - X_2|$. Can you find an unbiased estimator of n based on Z? Justify your answer. [4 marks]