5 Introduction to Probability (mj201)

(a) A travel agency is surveying their customer satisfaction by randomly polling 300 of their customers. From experience, 80% of their customers are typically happy with their service. Let $H$ be the number of happy customers in the current poll.

(i) Randomly polling 300 different customers, specify a suitable distribution for $H$, including its parameters, expected value and variance. [1 mark]

(ii) State a suitable approximation of $H$ and specify its distribution including its parameters, and compute the expected value and variance. [2 marks]

(iii) Using the approximation from Part (a)(ii), what is the probability that more than 220 and fewer that 260 customers are happy in the current poll? [4 marks]

(iv) Now, let $X$ be the proportion of customers that are happy in the current poll. Following your approximation from Part (a)(ii), give the distribution for $X$, including its parameters, expected value and variance. [3 marks]

(b) Let $X$ and $Y$ have a joint density function

$$f(x, y) = \begin{cases} \text{cx} & \text{if } 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the value of the constant $c > 0$. [4 marks]

(ii) Find the marginal density functions of $X$ and $Y$. [4 marks]

(iii) Are $X$ and $Y$ independent? Justify your answer. [2 marks]