

COMPUTER SCIENCE TRIPOS Part IA – 2021 – Paper 1

5 Introduction to Probability (mj201)

- (a) A travel agency is surveying their customer satisfaction by randomly polling 300 of their customers. From experience, 80% of their customers are typically happy with their service. Let H be the number of happy customers in the current poll.
- (i) Randomly polling 300 different customers, specify a suitable distribution for H , including its parameters, expected value and variance. [1 mark]
- (ii) State a suitable approximation of H and specify its distribution including its parameters, and compute the expected value and variance. [2 marks]
- (iii) Using the approximation from Part (a)(ii), what is the probability that more than 220 and fewer than 260 customers are happy in the current poll? [4 marks]
- (iv) Now, let X be the proportion of customers that are happy in the current poll. Following your approximation from Part (a)(ii), give the distribution for X , including its parameters, expected value and variance. [3 marks]
- (b) Let X and Y have a joint density function

$$f(x, y) = \begin{cases} cx & \text{if } 0 < y < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Find the value of the constant $c > 0$. [4 marks]
- (ii) Find the marginal density functions of X and Y . [4 marks]
- (iii) Are X and Y independent? Justify your answer. [2 marks]