8 Hoare Logic and Model Checking (jp622)

We consider the LTL temporal logic over atomic propositions $p \in \text{AP}$:

$$\phi \in \text{PathProp} ::= p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \phi_1 \lor \phi_2 \mid \phi_1 \rightarrow \phi_2 \mid X \phi \mid F \phi \mid G \phi \mid \phi_1 \mathbf{U} \phi_2.$$  

(a) Precisely state the semantics of the until operator $\phi_1 \mathbf{U} \phi_2$. [2 marks]

(b) Express $F \phi$ in terms of the until operator $- \mathbf{U} =$. [2 marks]

(c) Give models $\mathcal{M}_1, \mathcal{M}_2$ such that $\mathcal{M}_1 \models G (p \lor q)$ and $\mathcal{M}_2 \not\models (G p) \lor (G q)$, but either $\mathcal{M}_1 \not\models (G p) \lor (G q)$ or $\mathcal{M}_2 \not\models G (p \lor q)$ (indicate which). [3 marks]

(d) Starting from any strictly positive integer $n$, the transition system induced by going to $n/2$ if $n$ is even, and to $3 \times n + 1$ if $n$ is odd, is conjectured to always pass through $1$.

(i) Precisely describe this conjecture in the form of a model and an LTL formula. [4 marks]

(ii) Describe what shape a counterexample to this conjecture would have. [2 marks]

(e) Alice (a) and Bob (b) share a bicycle. To ensure they do not have problems, they have a protocol: they can express an interest for it (e), use it (u), or not need it (n), yielding atomic propositions $AP = \text{Pers} \times \text{Act}$, where $\text{Pers} ::= a \mid b$ and $\text{Act} ::= e \mid u \mid n$, so that for example a state labelled with $\{ae, bu\}$ is one where Alice has expressed interest in using the bike, and Bob has taken it.

Give LTL formulas for

(i) Alice does not keep the bike forever. [2 marks]

(ii) Non-starvation: if Alice expresses an interest in having the bike for long enough, she eventually gets it. [2 marks]

(iii) Alice cannot take the bike twice in a row if Bob expresses interest throughout. [3 marks]