8 Hoare Logic and Model Checking (jp622)

We consider the LTL temporal logic over atomic propositions $p \in \mathsf{AP}$:
$\phi \in \mathsf{PathProp} ::= p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \phi_1 \rightarrow \phi_2 \mid \mathsf{X}\ \phi \mid \mathsf{F}\ \phi \mid \mathsf{G}\ \phi \mid \phi_1\ \mathsf{U}\ \phi_2$.

(*a*) Precisely state the semantics of the until operator $\phi_1\ \mathsf{U}\ \phi_2$. [2 marks]

(*b*) Express $\mathsf{F}\ \phi$ in terms of the until operator $-\ \mathsf{U}\ =$. [2 marks]

(*c*) Give models $\mathcal{M}_1, \mathcal{M}_2$ such that $\mathcal{M}_1 \vDash \mathsf{G}\ (p \vee q)$ and $\mathcal{M}_2 \vDash (\mathsf{G}\ p) \vee (\mathsf{G}\ q)$, but either $\mathcal{M}_1 \nvDash (\mathsf{G}\ p) \vee (\mathsf{G}\ q)$ or $\mathcal{M}_2 \nvDash \mathsf{G}\ (p \vee q)$ (indicate which). [3 marks]

(*d*) Starting from any strictly positive integer n, the transition system induced by going to $n/2$ if n is even, and to $3 \times n + 1$ if n is odd, is conjectured to always pass through 1.

(*i*) Precisely describe this conjecture in the form of a model and an LTL formula. [4 marks]

(*ii*) Describe what shape a counterexample to this conjecture would have. [2 marks]

(*e*) Alice (a) and Bob (b) share a bicycle. To ensure they do not have problems, they have a protocol: they can express an interest for it (e), use it (u), or not need it (n), yielding atomic propositions $AP = Pers \times Act$, where $Pers ::= \mathsf{a} \mid \mathsf{b}$ and $Act ::= \mathsf{e} \mid \mathsf{u} \mid \mathsf{n}$, so that for example a state labelled with $\{\mathsf{ae}, \mathsf{bu}\}$ is one where Alice has expressed interest in using the bike, and Bob has taken it.

Give LTL formulas for

(*i*) Alice does not keep the bike forever. [2 marks]

(*ii*) Non-starvation: if Alice expresses an interest in having the bike for long enough, she eventually gets it. [2 marks]

(*iii*) Alice cannot take the bike twice in a row if Bob expresses interest throughout. [3 marks]