8 Hoare Logic and Model Checking (jp622)

We consider the LTL temporal logic over atomic propositions $p \in AP$:
\[
\phi \in \text{PathProp} ::= p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \phi_1 \lor \phi_2 \mid \phi_1 \rightarrow \phi_2 \mid X \phi \mid F \phi \mid G \phi \mid \phi_1 \U \phi_2.
\]

(a) Precisely state the semantics of the until operator $\phi_1 \U \phi_2$. [2 marks]

(b) Express $F \phi$ in terms of the until operator $- \U =$. [2 marks]

(c) Give models $M_1, M_2$ such that $M_1 \vDash (G p) \lor (G q)$ and $M_2 \vDash (G p) \lor (G q)$, but either $M_1 \not\vDash (G p) \lor (G q)$ or $M_2 \not\vDash (G p) \lor (G q)$ (indicate which). [3 marks]

(d) Starting from any strictly positive integer $n$, the transition system induced by going to $n/2$ if $n$ is even, and to $3 \times n + 1$ if $n$ is odd, is conjectured to always pass through 1.

(i) Precisely describe this conjecture in the form of a model and an LTL formula. [4 marks]

(ii) Describe what shape a counterexample to this conjecture would have. [2 marks]

(e) Alice (a) and Bob (b) share a bicycle. To ensure they do not have problems, they have a protocol: they can express an interest for it ($e$), use it ($u$), or not need it ($n$), yielding atomic propositions $AP = Pers \times Act$, where $Pers ::= a \mid b$ and $Act ::= e \mid u \mid n$, so that for example a state labelled with \{ae, bu\} is one where Alice has expressed interest in using the bike, and Bob has taken it.

Give LTL formulas for

(i) Alice does not keep the bike forever. [2 marks]

(ii) Non-starvation: if Alice expresses an interest in having the bike for long enough, she eventually gets it. [2 marks]

(iii) Alice cannot take the bike twice in a row if Bob expresses interest throughout. [3 marks]