7 Denotational Semantics (mpf23)

(a) (i) Define the notion of admissible subset of a domain and state Scott’s fixed point induction principle. [4 marks]

(ii) Let \((D, \sqsubseteq_D)\) and \((E, \sqsubseteq_E)\) be domains and let \(f : D \to E\) and \(g : E \to D\) be continuous functions.

Using Scott’s fixed point induction principle prove

(A) \(\text{fix}(f \circ g) \sqsubseteq_E f(\text{fix}(g \circ f))\)

(B) \(f(\text{fix}(g \circ f)) \sqsubseteq_E \text{fix}(f \circ g)\) [8 marks]

(b) (i) Define the contextual-equivalence relation \(P_1 \cong_{\text{ctx}} P_2 : \tau\) for pairs of closed PCF expressions \(P_1, P_2\) and a PCF type \(\tau\). [2 marks]

(ii) Prove or disprove the following statement.

For every pair of PCF types \(\sigma, \tau\) and every pair of closed PCF expressions \(M\) of type \(\sigma \to \tau\) and \(N\) of type \(\tau \to \sigma\),

\[
\text{fix}(\text{fn} \ y : \tau. M(N(y))) \cong_{\text{ctx}} M(\text{fix}(\text{fn} \ x : \sigma. N(M(x)))) : \tau
\]

[6 marks]