7 Denotational Semantics (mpf23)

(*a*) (*i*) Define the notion of admissible subset of a domain and state Scott's fixed point induction principle. [4 marks]

(*ii*) Let $(D, \sqsubseteq_D)$ and $(E, \sqsubseteq_E)$ be domains and let $f : D \to E$ and $g : E \to D$ be continuous functions.

Using Scott's fixed point induction principle prove

(A) $\mathit{fix}(f \circ g) \sqsubseteq_E f\big(\mathit{fix}(g \circ f)\big)$

(B) $f\big(\mathit{fix}(g \circ f)\big) \sqsubseteq_E \mathit{fix}(f \circ g)$

[8 marks]

(*b*) (*i*) Define the contextual-equivalence relation $P_1 \cong_{\mathrm{ctx}} P_2 : \tau$ for pairs of closed PCF expressions P_1, P_2 and a PCF type τ. [2 marks]

(*ii*) Prove or disprove the following statement.

For every pair of PCF types σ, τ and every pair of closed PCF expressions M of type $\sigma \to \tau$ and N of type $\tau \to \sigma$,

$$\mathbf{fix}\big(\mathbf{fn}\ y : \tau.\ M(N(y))\big) \cong_{\mathrm{ctx}} M\Big(\mathbf{fix}\big(\mathbf{fn}\ x : \sigma.\ N(M(x))\big)\Big) : \tau$$

[6 marks]