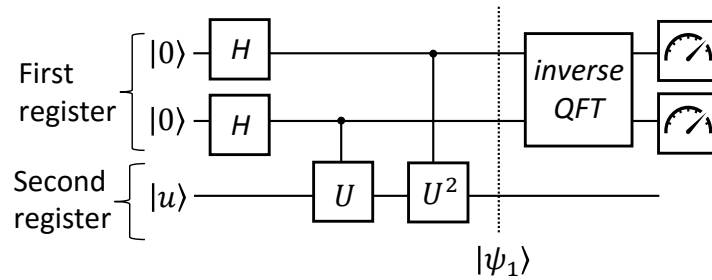


14 Quantum Computing (sjh227)



- (a) Show that $U = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ is unitary. [1 mark]
- (b) Show that $\begin{bmatrix} 1 + \sqrt{5} \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 - \sqrt{5} \\ 2 \end{bmatrix}$ are (un-normalised) eigenvectors of U , and give the eigenvalues. [2 marks]
- (c) The figure shows the quantum circuit for quantum phase estimation to two bits of precision for a single-qubit unitary. Quantum phase estimation is performed to two bits of precision with $U = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $|u\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, find the quantum state, $|\psi_1\rangle$, before the inverse quantum Fourier transform is executed. [7 marks]
- (d) What possible measurement outcomes can occur (i.e., after the inverse quantum Fourier transform, with measurement in the computational basis)? Give probabilities for each possible outcome. [6 marks]
- (e) Give two applications of quantum phase estimation, and for each give the state, $|u\rangle$, in which the second register should be prepared, and briefly outline how these can be prepared in practise. [4 marks]