## COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 9

## 10 Machine Learning and Bayesian Inference (sbh11)

The central limit theorem tells us that, if  $X_i$  are random variables,  $\mu$  is the mean of  $X_i$  and  $\sigma^2$  is the variance of  $X_i$  then, under suitable conditions

$$\frac{\hat{X}_n - \mu}{\sigma/\sqrt{n}} \to N(0, 1)$$

where N(0,1) denotes the normal density with mean 0 and variance 1, and

$$\hat{X}_n = \frac{1}{n} \sum_{i=1}^n X_i.$$

(a) Let Y have density N(0,1). We know that, for a parameter p, there is a constant  $z_p$  such that

$$\Pr(-z_p \le Y \le z_p) > p.$$

Show that with probability at least p, the quantity  $\mu$  as defined above is in the interval described by  $\hat{X}_n \pm z_p(\sigma/\sqrt{n})$ . [3 marks]

(b) The quantity  $\hat{X}_n$  can be regarded as an estimate of  $\mu$ . If the random variables  $X_i$  take values in  $\{0,1\}$ , and are also independent and identically distributed, explain why it might make sense to estimate  $\sigma^2$  as

$$\sigma^2 \simeq s = \hat{X}_n (1 - \hat{X}_n).$$

[3 marks]

- (c) Define what it means for an estimate such as that suggested in Part (b) to be unbiased. Is the estimate suggested in Part (b) unbiased? Provide a proof of your answer. [7 marks]
- (d) We have two binary classifiers  $h_1$  and  $h_2$ , and a test set **s** containing 1000 examples. During testing,  $h_1$  makes 105 errors and  $h_2$  makes 120 errors. Explain how we can estimate a confidence interval of the kind defined in Part (a) for the difference  $(\operatorname{er}(h_1) \operatorname{er}(h_2))$  between the true error probabilities  $\operatorname{er}(h_1)$  and  $\operatorname{er}(h_2)$  of the classifiers. Your answer should be careful to state any assumptions or approximations being made.