COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 9

1 Advanced Algorithms (tms41)

(*a*) (*i*) What is the approximation ratio of an approximation algorithm? [2 marks]

(*ii*) State the definitions of PTAS and FPTAS. [4 marks]

(*b*) Consider the two approximation algorithms for VERTEX-COVER from the lectures (one greedy algorithm and one based on rounding a linear program).

(*i*) What are the approximation ratios of these two algorithms? [2 marks]

(*ii*) Construct an input graph that demonstrates the tightness of the approximation ratio of the greedy algorithm (for full marks, your construction should work for any even number of vertices n). [3 marks]

(*c*) Consider the following randomised algorithm to compute a solution of the VERTEX-COVER problem for an unweighted graph $G = (V, E)$:

```
Let C be the empty set
While E not empty do
       Pick any edge e={u,v} from E
       Choose x from {u,v} uniformly at random
       Add x to C
       Remove all edges incident to x from E
End While
Return C
```

(*i*) Explain briefly why the set C returned is a valid vertex cover. [2 marks]

(*ii*) Find a lower bound on the probability that the algorithms returns an optimal solution.
Hint: For each edge $e = \{u, v\}$ picked by the algorithm consider the event that the chosen vertex $x \in \{u, v\}$ added to C is also part of an optimal cover. [4 marks]

(*iii*) Given a lower bound $p \in (0, 1)$ on the probability that this algorithm returns an optimal solution, describe a new algorithm that returns an optimal solution with probability at least δ, for any given $\delta \in [p, 1)$. [3 marks]