

1 Advanced Algorithms (tms41)

- (a) (i) What is the approximation ratio of an approximation algorithm? [2 marks]
- (ii) State the definitions of PTAS and FPTAS. [4 marks]
- (b) Consider the two approximation algorithms for VERTEX-COVER from the lectures (one greedy algorithm and one based on rounding a linear program).
- (i) What are the approximation ratios of these two algorithms? [2 marks]
- (ii) Construct an input graph that demonstrates the tightness of the approximation ratio of the greedy algorithm (for full marks, your construction should work for any even number of vertices n). [3 marks]
- (c) Consider the following randomised algorithm to compute a solution of the VERTEX-COVER problem for an unweighted graph $G = (V, E)$:

```
Let C be the empty set
While E not empty do
    Pick any edge e={u,v} from E
    Choose x from {u,v} uniformly at random
    Add x to C
    Remove all edges incident to x from E
End While
Return C
```

- (i) Explain briefly why the set C returned is a valid vertex cover. [2 marks]
- (ii) Find a lower bound on the probability that the algorithm returns an optimal solution.
Hint: For each edge $e = \{u, v\}$ picked by the algorithm consider the event that the chosen vertex $x \in \{u, v\}$ added to C is also part of an optimal cover. [4 marks]
- (iii) Given a lower bound $p \in (0, 1)$ on the probability that this algorithm returns an optimal solution, describe a new algorithm that returns an optimal solution with probability at least δ , for any given $\delta \in [p, 1)$. [3 marks]