1 Advanced Algorithms (tms41)

(a) (i) What is the approximation ratio of an approximation algorithm? [2 marks]

(ii) State the definitions of PTAS and FPTAS. [4 marks]

(b) Consider the two approximation algorithms for VERTEX-COVER from the lectures (one greedy algorithm and one based on rounding a linear program).

(i) What are the approximation ratios of these two algorithms? [2 marks]

(ii) Construct an input graph that demonstrates the tightness of the approximation ratio of the greedy algorithm (for full marks, your construction should work for any even number of vertices $n$). [3 marks]

(c) Consider the following randomised algorithm to compute a solution of the VERTEX-COVER problem for an unweighted graph $G = (V, E)$:

Let $C$ be the empty set
While $E$ not empty do
    Pick any edge $e = \{u, v\}$ from $E$
    Choose $x$ from $\{u, v\}$ uniformly at random
    Add $x$ to $C$
    Remove all edges incident to $x$ from $E$
End While
Return $C$

(i) Explain briefly why the set $C$ returned is a valid vertex cover. [2 marks]

(ii) Find a lower bound on the probability that the algorithms returns an optimal solution. 
    Hint: For each edge $e = \{u, v\}$ picked by the algorithm consider the event that the chosen vertex $x \in \{u, v\}$ added to $C$ is also part of an optimal cover. [4 marks]

(iii) Given a lower bound $p \in (0, 1)$ on the probability that this algorithm returns an optimal solution, describe a new algorithm that returns an optimal solution with probability at least $\delta$, for any given $\delta \in [p, 1)$. [3 marks]