COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 8

6 Denotational Semantics (mpf23)

(a) For a poset (P, \sqsubseteq) , the join of $x, y \in P$ is defined to be the element $x \sqcup y \in P$ such that

 $\forall \ p \in P. \ x \sqcup y \sqsubseteq p \iff (x \sqsubseteq p \land y \sqsubseteq p)$

A poset is said to be *join complete* if every pair of elements in it has a join.

For a join-complete cpo D, show that the function $\sqcup : D \times D \to D$ mapping $(x, y) \in D \times D$ to $x \sqcup y \in D$ is continuous. [8 marks]

- (b) Let (D, \sqsubseteq) be a domain.
 - (i) For a continuous function $f: D \to D$, prove that the subset of D

$$\widehat{f} = \{ x \in D \mid f(x) \sqsubseteq x \}$$

ordered by \sqsubseteq is a domain.

(*ii*) For $d \in D$, let $\uparrow(d) = \{ x \in D \mid d \sqsubseteq x \}$.

For a continuous function $g: D \to D$, prove that if (D, \sqsubseteq) is join complete then, for all $d \in D$, the subset of D

$$\uparrow(d) \cap \widehat{g}$$

ordered by \sqsubseteq is a domain.

[6 marks]

[6 marks]