COMPUTER SCIENCE TRIPOS Part II – 2020 – Paper 8

15 Types (nk480)

(*a*) Recall that in constructive logic, logical negation is defined using implication and falsehood as $\neg A \triangleq A \supset \bot$.

(*i*) Does $A \supset \neg\neg A$? If so, give a simply-typed lambda-term corresponding to this implication.

(*ii*) Does $\neg\neg A \supset A$? If so, give a simply-typed lambda-term corresponding to this implication.

(*iii*) Does $\neg\neg\neg A \supset \neg A$? If so, give a simply-typed lambda-term corresponding to this implication.

[5 marks]

(*b*) (*i*) Give the typing rules for Peano natural numbers and their eliminator. [2 marks]

(*ii*) Using the rules given above, define the addition function. [3 marks]

(*iii*) Let a binary tree be either a leaf `Leaf` or a node `Node(l,x,r)` where `l` and `r` are subtrees, and `x` is a natural number. Give typing rules for trees corresponding to this prose description, including an eliminator. [3 marks]

(*iv*) Using the rules given above, define a function `size` which takes a binary tree and returns the total number of nodes in the tree. [5 marks]

(*c*) The `zip` function takes two lists, and returns a list of pairs of the elements as output. Suppose we see the following Agda type declaration for `zip`:

$$zip : \forall\{A\,B : Set\} \to \{n : Nat\} \to \mathsf{Vec}\,A\,n \to \mathsf{Vec}\,B\,n \to \mathsf{Vec}\,(A \times B)\,n$$

Explain what this means in terms of how to call the function, and what properties the result has. [2 marks]