1 Advanced Algorithms (tms41)

(*a*) State the fundamental theorem of Linear Programming. [3 marks]

(*b*) Consider the following linear program:

$$\begin{aligned} \text{minimise} \quad & 4 \cdot x_1 - x_2 \\ -x_1 + 5x_2 &\geq 4 \\ x_1 - 0.5x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(*i*) Convert this linear program into slack form. [3 marks]

(*ii*) What is the number of different slack forms of the linear program in Part (*b*)(*i*)? [2 marks]

(*iii*) Give at least one non-feasible and one feasible basic solution of the linear program in (*b*)(*i*). [4 marks]

(*c*) Consider the following separation problem. We are given m points $x^1 = (x_1^1, x_2^1), x^2 = (x_1^2, x_2^2), \ldots, x^m = (x_1^m, x_2^m) \in \mathbb{R}^2$ and n points $y^1 = (y_1^1, y_2^1), y^2 = (y_1^2, y_2^2), \ldots, y^n = (y_1^n, y_2^n) \in \mathbb{R}^2$. The goal is to find a "separating" vector $w = (w_1, w_2) \in \mathbb{R}^2$ (if it exists) such that:

$$\langle x^i, w \rangle = \sum_{j=1}^{2} x_j^i w_j > 0 \qquad \text{for } i = 1, 2, ..., m,$$

and

$$\langle y^i, w \rangle = \sum_{j=1}^{2} y_j^i w_j < 0 \qquad \text{for } i = 1, 2, ..., n.$$

(*i*) Create a new, equivalent system of inequalities by replacing each strict inequality by a suitable non-strict inequality. Justify why this new system has a solution if and only if the original system has one. [4 marks]

(*ii*) Based on your answer in Part (*c*)(*i*), how can you solve the above problem using linear programming? [4 marks]