9  Logic and Proof (lp15)

(a) In the context of automatic theorem proving, provide one-sentence definitions of each of the following concepts: satisfiable, sound, complete. You may take as given the definitions of all underlying concepts. [3 marks]

(b) Mordred has written a resolution theorem prover, but there are bugs in his code. Very rarely, one of the following errors occurs: a literal is deleted from a clause; an entire clause is deleted; the “occurs check” of unification is not performed. Briefly describe, with justification, the consequences of each type of error. [3 marks]

(c) For each of the following sets of clauses, either derive the empty clause or demonstrate that the set is satisfiable by exhibiting a model. Below, $a$ and $b$ are constants, while $x$, $y$ and $z$ are variables.

(i) \[
\begin{align*}
\{ R(a) \} & \quad \{ \neg R(x), \neg Q(f(x)), \neg R(a) \} \\
\{ Q(z), P(z) \} & \quad \{ \neg P(y), \neg R(y) \}
\end{align*}
\] [7 marks]

(ii) \[
\begin{align*}
\{ R(a), R(b) \} & \quad \{ \neg R(x), \neg Q(f(x)), \neg R(y) \} \\
\{ Q(x), \neg P(y) \} & \quad \{ Q(z), P(z) \}
\end{align*}
\] [7 marks]