6 Computation Theory (amp12)

(*a*) Define the Church numerals for zero ($\underline{0}$), one ($\underline{1}$) and for an arbitrary natural number ($\underline{n}$). [2 marks]

(*b*) Define encodings of Booleans as λ-terms (True, False and If). [1 mark]

(*c*) Explain what it means for a λ term to *represent* a number-valued partial function of n numerical arguments; do the same for one returning Boolean instead of numerical results. [3 marks]

(*d*) Give λ-terms that represent the following functions:

(*i*) successor (Succ) [1 mark]

(*ii*) test for zero (Eq_0) [1 mark]

(*e*) Define encodings of pairing and projections (Pair, Fst and Snd). [2 marks]

(*f*) What function $\mathbb{N} \to \mathbb{N}$ is represented by the following λ-term? Carefully justify your answer.

$$\lambda x.\, \mathsf{Snd}(x(\lambda y.\, \mathsf{Pair}(\mathsf{Succ}(\mathsf{Fst}\, y))(\mathsf{Fst}\, y))(\mathsf{Pair}\, \underline{0}\, \underline{0}))$$

[6 marks]

(*g*) Give with justification a λ-term that represents the function mapping each pair of numbers (m, n) to True if $m \leq n$ and to False otherwise. [Hint: use the λ-terms from parts $(d)(ii)$ and (f).] [4 marks]