

5 Computation Theory (amp12)

- (a) Explain what it means for a partial function $f : \mathbb{N}^n \rightarrow \mathbb{N}$ to be register machine *computable* and for a set of numbers $S \subseteq \mathbb{N}$ to be register machine *decidable*. [5 marks]
- (b) A set of numbers $S \subseteq \mathbb{N}$ is register machine *enumerable* if either S is empty, or $S = \{f(n) \mid n \in \mathbb{N}\}$ for some total function $f : \mathbb{N} \rightarrow \mathbb{N}$ which is register machine computable.
- (i) Show that if S is register machine decidable, then it is register machine enumerable. [5 marks]
- (ii) Show that if both S and its complement $\bar{S} \triangleq \{n \in \mathbb{N} \mid n \notin S\}$ are register machine enumerable, then S is register machine decidable. [5 marks]
- (iii) Give an example of a set of numbers that is register machine enumerable, but not register machine decidable. (Any standard results about computable functions that you use should be carefully stated.) [5 marks]