3 Complexity Theory (ad260)

Recall that, for languages A and B, $A \leq_P B$ denotes that A is *polynomial-time reducible* to B.

(*a*) Show that if $A \leq_P B$ and $B \leq_P C$, then $A \leq_P C$. [3 marks]

(*b*) Show that if A is NP-complete and $A \leq_P B$ and $B \leq_P A$, then B is also NP-complete. [3 marks]

For any $k \in \mathbb{N}$, let k-EC (*exact cover by k sets*) denote the language coding the following decision problem.

> Given a set X and a collection S of k-element subsets of X, determine if there exists a set $C \subseteq S$ such that each $x \in X$ appears in exactly one element of C.

(*c*) Show that, for each fixed k, k-EC $\leq_P$ $(k+1)$-EC. [5 marks]

For each of the following statements, determine whether the it is true, false or unknown and give full justification for your answer. If the answer is unknown, the justification may be that it is equivalent to some well-known open problem. You may assume standard facts about the complexity of matching and covering problems covered in the lectures, as long as you state them clearly.

(*d*) 2-EC $\leq_P$ 4-EC [3 marks]

(*e*) 4-EC $\leq_P$ 3-EC [3 marks]

(*f*) 4-EC $\leq_P$ 2-EC [3 marks]