10 Logic and Proof (lp15)

(*a*) From n distinct propositional letters, each of which may be negated or not, 2^n distinct clauses can be created. Present a satisfying interpretation of this set of 2^n clauses or demonstrate that none exists. [3 marks]

(*b*) Sketch the operation of the DPLL algorithm when provided with the set of clauses described above, including an estimate of its time complexity as a function of n. [4 marks]

(*c*) For each of the following formulas, present either a proof in the sequent calculus, or a falsifying interpretation. The modal logic is S4.

(*i*) $\Box(P \vee Q) \to (\Box\Diamond\neg P \to \Diamond\Box Q)$ [5 marks]

(*ii*) $\exists x\, P(f(x)) \wedge \forall x\, [P(x) \to Q(g(x))] \to \exists y\, Q(y)$ [4 marks]

(*iii*) $\exists x\, (P(x) \to Q(x)) \to [\exists x\, P(x) \to \exists x\, Q(x)]$ [4 marks]