1 Artificial Intelligence (sbh11)

A neural network takes input vectors $x \in \mathbb{R}^n$, has a single layer of hidden nodes $h_i$ where $i = 1, \ldots, p$, and a single output node $o$.

All nodes compute the function $z = \sigma(a)$ where

$$a = \sum_{i=1}^{m} w_i z_i + w_0.$$  

Here $z$ and $z_i$ denote the inputs and outputs of the node, and each node has its own set of weights $w_0, w_1, \ldots, w_m$. Examples take the form $(x, y)$ and the error the network makes for an example is $E(x, y, w)$, where $w$ is the collection of all the weights in the network.

(a) An example has been applied to the network and we know the quantity $\delta = \partial E / \partial a$ for the output node $o$. Explain how this knowledge can be used to compute the partial derivative of $E$ with respect to the weights for some hidden node $h_i$. [8 marks]

(b) The hidden nodes are now replaced with a different type of node, computing

$$z_i = h_i(x) = \phi(||x - c_i||^2)$$

where $\phi$ is some new function, the $c_i \in \mathbb{R}^n$ are the parameters for the new nodes, and

$$||v||^2 = v^T v.$$

Give a detailed derivation of a training algorithm for this network. [12 marks]