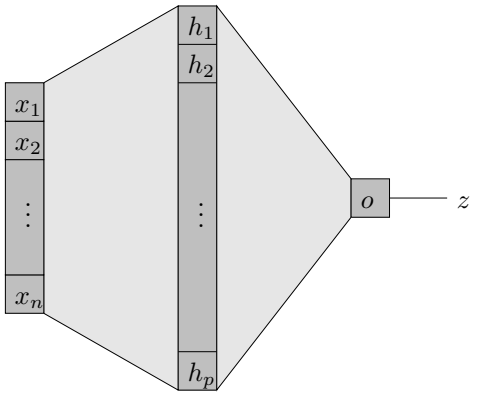
COMPUTER SCIENCE TRIPOS Part IB – 2020 – Paper 6

1 Artificial Intelligence (sbh11)

A neural network takes input vectors $\mathbf{x} \in \mathbb{R}^n$, has a single layer of hidden nodes h_i where $i = 1, \ldots, p$, and a single output node o.



All nodes compute the function $z = \sigma(a)$ where

$$a = \sum_{i=1}^{m} w_i z_i + w_0.$$

Here z and z_i denote the inputs and outputs of the node, and each node has its own set of weights $w_0, w_1, \ldots, w_m$. Examples take the form $(\mathbf{x}, y)$ and the error the network makes for an example is $E(\mathbf{x}, y, \mathbf{w})$, where $\mathbf{w}$ is the collection of all the weights in the network.

(*a*) An example has been applied to the network and we know the quantity $\delta = \partial E / \partial a$ for the output node o. Explain how this knowledge can be used to compute the partial derivative of E with respect to the weights for some hidden node h_i. [8 marks]

(*b*) The hidden nodes are now replaced with a different type of node, computing

$$z_i = h_i(\mathbf{x}) = \phi(||\mathbf{x} - \mathbf{c}_i||^2)$$

where ϕ is some new function, the $\mathbf{c}_i \in \mathbb{R}^n$ are the parameters for the new nodes, and

$$||\mathbf{v}||^2 = \mathbf{v}^T \mathbf{v}.$$

Give a detailed derivation of a training algorithm for this network. [12 marks]