

COMPUTER SCIENCE TRIPOS Part IA – 2020 – Paper 2

7 Discrete Mathematics (mpf23)

(a) Prove that, for all statements  $P$  and  $Q$ ,

$$(P \implies Q) \implies ((P \implies \neg Q) \implies \neg P)$$

[4 marks]

(b) (i) Let  $p$  and  $q$  be positive integers such that  $\gcd(p, q) = 1$ .

Prove that, for all integers  $a$  and  $b$ ,

$$a \equiv b \pmod{p \cdot q} \iff (a \equiv b \pmod{p} \wedge a \equiv b \pmod{q})$$

[5 marks]

(ii) State Fermat's Little Theorem.

[3 marks]

(iii) Let  $p$  and  $q$  be distinct prime numbers and let  $e$  and  $d$  be natural numbers such that  $e \cdot d \equiv 1 \pmod{(p-1) \cdot (q-1)}$ .

Prove that, for all natural numbers  $n$ ,

$$n^{e \cdot d} \equiv n \pmod{p \cdot q}$$

[8 marks]