

10 Discrete Mathematics (fms27)

- (a) Draw the state transition diagram of a deterministic finite automaton (DFA) that accepts language

$$L_1 = \left\{ x \in \{0, 1\}^* \mid \begin{array}{l} x \text{ is divisible by 8 if interpreted} \\ \text{as an unsigned binary integer} \end{array} \right\}.$$

Explain your construction. [6 marks]

- (b) State whether

$$L_2 = \{ x \in \{\oplus, \ominus\}^* \mid \text{no left substring of } x \text{ has more } \ominus\text{s than } \oplus\text{s} \}$$

(the “never in debt” language) is a regular language or not. Prove your answer. [8 marks]

- (c) Consider language L_3 over the $\{A, B, C\}$ alphabet, defined by the following inductive rules.

$$\frac{}{AB} \text{ (0)} \quad \frac{Ax}{Axx} \text{ (1)} \quad \frac{xBBBy}{xCy} \text{ (2)} \quad \frac{xCCy}{xy} \text{ (3)} \quad \frac{xB}{xBC} \text{ (4)}$$

- (i) Produce three distinct derivations for string $ABB \in L_3$. [3 marks]

- (ii) Argue why $AC \notin L_3$. [*Hint*: This is a difficult challenge and therefore even a good insight, rather than a full proof, might earn full marks.] [3 marks]