10 Discrete Mathematics (fms27)

(*a*) Draw the state transition diagram of a deterministic finite automaton (DFA) that accepts language

$$L_1 = \left\{ x \in \{0,1\}^* \;\middle|\; \begin{array}{l} x \text{ is divisible by 8 if interpreted} \\ \text{as an unsigned binary integer} \end{array} \right\}.$$

Explain your construction. [6 marks]

(*b*) State whether

$$L_2 = \left\{ x \in \{\oplus, \ominus\}^* \;\middle|\; \text{no left substring of } x \text{ has more } \ominus\text{s than } \oplus\text{s} \right\}$$

(the "never in debt" language) is a regular language or not. Prove your answer. [8 marks]

(*c*) Consider language L_3 over the {A, B, C} alphabet, defined by the following inductive rules.

$$\frac{}{\mathtt{AB}}\ (0) \qquad \frac{\mathtt{A}x}{\mathtt{A}xx}\ (1) \qquad \frac{x\mathtt{BBB}y}{x\mathtt{C}y}\ (2) \qquad \frac{x\mathtt{CC}y}{xy}\ (3) \qquad \frac{x\mathtt{B}}{x\mathtt{BC}}\ (4)$$

(*i*) Produce three distinct derivations for string $\mathtt{ABB} \in L_3$. [3 marks]

(*ii*) Argue why $\mathtt{AC} \notin L_3$. [*Hint:* This is a difficult challenge and therefore even a good insight, rather than a full proof, might earn full marks.] [3 marks]