9 Algorithms (djw1005)

We are given a directed graph $g = (V, E)$. A vertex $v \in V$ is said to be an origin if for any other vertex $w \in V$ there is a directed path from $v$ to $w$.

(a) Consider the $\text{dfs}\_\text{recurse}(g, s)$ algorithm as described in lecture notes. Show carefully that, once it terminates, if it has visited a vertex $v$ then it has also visited all vertices reachable from $v$. [4 marks]

(b) Suppose $g$ has an origin. Give an algorithm that returns an origin, and which has $O(V + E)$ running time. [Hint: Consider $\text{dfs}\_\text{recurse}\_\text{all}(g)$. What happens after it visits an origin?] [5 marks]

(c) Suppose $g$ has an origin. Prove that the vertex returned by your algorithm in part (b) is indeed an origin. [6 marks]

(d) Give an algorithm that returns all origins, and which has $O(V + E)$ running time. If the graph has no origins, your algorithm should return an empty set. Explain briefly why your algorithm is correct. [5 marks]