9 Algorithms (djw1005)

We are given a directed graph \( g = (V, E) \). A vertex \( v \in V \) is said to be an *origin* if for any other vertex \( w \in V \) there is a directed path from \( v \) to \( w \).

(a) Consider the \texttt{dfs_recurse}(g, s) algorithm as described in lecture notes. Show carefully that, once it terminates, if it has visited a vertex \( v \) then it has also visited all vertices reachable from \( v \). \([4 \text{ marks}]\)

(b) Suppose \( g \) has an origin. Give an algorithm that returns an origin, and which has \( O(V + E) \) running time. \([\text{Hint: Consider } \texttt{dfs_recurse_all}(g) \text{. What happens after it visits an origin?}]\) \([5 \text{ marks}]\)

(c) Suppose \( g \) has an origin. Prove that the vertex returned by your algorithm in part (b) is indeed an origin. \([6 \text{ marks}]\)

(d) Give an algorithm that returns *all* origins, and which has \( O(V + E) \) running time. If the graph has no origins, your algorithm should return an empty set. Explain briefly why your algorithm is correct. \([5 \text{ marks}]\)