9   Algorithms (djw1005)

We are given a directed graph $g = (V, E)$. A vertex $v \in V$ is said to be an *origin* if for any other vertex $w \in V$ there is a directed path from v to w.

(*a*) Consider the `dfs_recurse(`g`,`s`)` algorithm as described in lecture notes. Show carefully that, once it terminates, if it has visited a vertex v then it has also visited all vertices reachable from v.   [4 marks]

(*b*) Suppose g has an origin. Give an algorithm that returns an origin, and which has $O(V + E)$ running time. [*Hint:* Consider `dfs_recurse_all(`g`)`. What happens after it visits an origin?]   [5 marks]

(*c*) Suppose g has an origin. Prove that the vertex returned by your algorithm in part (*b*) is indeed an origin.   [6 marks]

(*d*) Give an algorithm that returns *all* origins, and which has $O(V + E)$ running time. If the graph has no origins, your algorithm should return an empty set. Explain briefly why your algorithm is correct.   [5 marks]