COMPUTER SCIENCE TRIPOS Part IA – 2020 – Paper 1

7 Algorithms (fms27)

Consider binary trees whose nodes have three fields: key (a single character between U and Z), left subtree, right subtree, with the two subtrees either both empty or both non-empty. Assuming suitable constructors, indicate an empty tree as T() and a non-empty tree as T(key, leftSubtree, rightSubtree).

(a) Define unambiguous pre-order, in-order and post-order representations for such a tree t, called r_{pre}(t), r_{in}(t), r_{post}(t), consisting of strings over the {U, V, W, X, Y, Z, (,)} alphabet, with balanced brackets, with three characters (two brackets and a letter) for each node, starting and finishing with a bracket unless t is empty. Formally describe your three representations for a generic tree t, then produce the corresponding strings for the following tree. [6 marks]



(b) In this obfuscated pseudocode, the input v0 is a syntactically correct $\mathbf{r}_{post}(t)$. Clearly explain (i) the purpose of the code; (ii) how it works; and (iii) how one should invoke it. Substitute meaningful explanatory identifiers for those T_x , v_y and \mathbf{m}_z . Identifiers v4 and v6 are worth more marks than the others. [7 marks]

```
class T1:
0
       # data members of objects of type T1
 1
 2
        v2: T14 of T3
       v4: T12
 3
        v6: T13
 4
5
       def m5(v0):
 6
           if v0 is "":
 7
               return T3()
 8
9
            else:
               for v9 in v0:
10
                   m10(v9)
11
               return v2.pop()
12
13
        def m10(v11):
14
           if v11 is one of {"U", "V", "W", "X", "Y", "Z"}:
15
               v6 = v4 is ")"
16
            else if v11 is ")":
17
18
               if v6:
                   v8 = v2.pop(); v7 = v2.pop()
19
20
               else:
                   v8 = null; v7 = null
21
               v2.push(T3(v4, v7, v8))
22
23
            v4 = v11
```

- (c) Explain in detail (i) how line 16 works, and (ii) why v4 will never be uninitialised when line 16 is executed. [4 marks]
- (d) Write clear pseudocode that takes a Tree t and returns $r_{in}(t)$. [3 marks]