

COMPUTER SCIENCE TRIPOS Part IA – 2020 – Paper 1

6 Introduction to Probability (tms41)

(a) Give the probability mass function of each of the three distributions:

(i) Poisson distribution, [2 marks]

(ii) Bernoulli distribution, [2 marks]

(iii) Binomial distribution. [2 marks]

(b) The football association has asked you to analyse the England team football matches from previous big tournaments. For each of the three situations below, choose a suitable distribution and compute its expectation and variance.

*Note:* In Part (b)(ii) you do **not** have to compute explicit numerical values.

(i) You analyse 2000 penalty kicks from the last 10 years of big tournaments. It turns out that 1200 of those 2000 penalty kicks were goals. A penalty kick is chosen at random. Let  $X$  be a success if a goal is scored. [2 marks]

(ii) Consider again the setting from (b)(i). If you pick 50 penalty kicks without replacement, let  $Y$  be the number of missed goals out of that sample. [2 marks]

(iii) Taking into account all games from the last 10 years of big tournaments, the England football team scored an average of 1 goal every 30 minutes. Let  $Z$  be the number of scored goals during a match of 90 minutes. [2 marks]

(c) Consider the following table displaying the joint distribution of two random variables  $X$  and  $Y$ .

	$x$			
$y$	-1	0	+1	$\mathbf{P}[Y = y]$
-1	??	??	0	$\frac{1}{4}$
0	$\frac{1}{4}$	??	??	$\frac{1}{2}$
+1	??	$\frac{1}{4}$	??	$\frac{1}{4}$
$\mathbf{P}[X = x]$	$\frac{1}{4}$	$\frac{1}{2}$	??	

(i) Complete the table above. [2 marks]

(ii) Compute  $\mathbf{E}[X]$  and  $\mathbf{E}[Y]$ . [2 marks]

(iii) Define independence of two discrete random variables. Are  $X$  and  $Y$  given above independent? Justify your answer. [2 marks]

(iv) Define the covariance of two random variables. What is the covariance between  $X$  and  $Y$  given above? Justify your answer. [2 marks]