6 Introduction to Probability (tms41)

(*a*) Give the probability mass function of each of the three distributions:

(*i*) Poisson distribution, [2 marks]

(*ii*) Bernoulli distribution, [2 marks]

(*iii*) Binomial distribution. [2 marks]

(*b*) The football association has asked you to analyse the England team football matches from previous big tournaments. For each of the three situations below, choose a suitable distribution and compute its expectation and variance.
Note: In Part (*b*)(*ii*) you do **not** have to compute explicit numerical values.

(*i*) You analyse 2000 penalty kicks from the last 10 years of big tournaments. It turns out that 1200 of those 2000 penalty kicks were goals. A penalty kick is chosen at random. Let X be a success if a goal is scored. [2 marks]

(*ii*) Consider again the setting from (*b*)(*i*). If you pick 50 penalty kicks without replacement, let Y be the number of missed goals out of that sample. [2 marks]

(*iii*) Taking into account all games from the last 10 years of big tournaments, the England football team scored an average of 1 goal every 30 minutes. Let Z be the number of scored goals during a match of 90 minutes. [2 marks]

(*c*) Consider the following table displaying the joint distribution of two random variables X and Y.

	x			
y	-1	0	$+1$	$\mathbf{P}[Y=y]$
-1	??	??	0	$\frac{1}{4}$
0	$\frac{1}{4}$	??	??	$\frac{1}{2}$
$+1$	??	$\frac{1}{4}$	??	$\frac{1}{4}$
$\mathbf{P}[X=x]$	$\frac{1}{4}$	$\frac{1}{2}$	??	

(*i*) Complete the table above. [2 marks]

(*ii*) Compute $\mathbf{E}[X]$ and $\mathbf{E}[Y]$. [2 marks]

(*iii*) Define independence of two discrete random variables. Are X and Y given above independent? Justify your answer. [2 marks]

(*iv*) Define the covariance of two random variables. What is the covariance between X and Y given above? Justify your answer. [2 marks]