5 Introduction to Probability (mj201)

(*a*) You are looking forward to the European Cup 2020. The tournament is going to last for 30 days. Each day during the tournament, you want to invite all of your 100 classmates to your house. But people might be busy on any given day, so you expect each classmate to come with probability 0.03 on each day, and they show up independently of one another. Let X denote the number of classmates that show up on any given day.
Note: In parts (*ii*), (*iii*) and (*iv*) you do **not** have to compute explicit numerical values.

(*i*) Give the exact and approximate distributions of X along with parameters. Explain why the approximation is valid. [3 marks]

(*ii*) What is the approximate probability that between 2 and 4 classmates, inclusive, show up on any given day? [3 marks]

(*iii*) What is the exact probability that at least 2 classmates show up on any given day? [3 marks]

(*iv*) What is the probability that there will be more than 27 days where at least 2 classmates show up? [3 marks]

(*b*) Suppose that each classmate is asked to arrive at 8pm, but the actual arrival time differs in minutes by a continuous uniform distribution $[-\theta, +\theta]$, where θ is an unknown parameter. Your data set $Y_1, Y_2, \ldots, Y_k$ is a realisation of independent random samples from that distribution, for some integer $k \geq 1$.

(*i*) Show that

$$T = \frac{3}{k} \cdot (Y_1^2 + Y_2^2 + \cdots + Y_k^2)$$

is an unbiased estimator for θ^2. [3 marks]

(*ii*) Is $\sqrt{T}$ also an unbiased estimator for θ? [1 mark]

(*iii*) Justify your answer in (*b*)(*ii*). [4 marks]