This question is about modelling a program, defined below, consisting of two threads and a single (mathematical) integer variable \(X\), initially set to 0. Each thread \(t\) has its own program counter given by \(pc_t\), initially set to 0, which describes the current line for that thread.

\[
\begin{align*}
\text{Thread 1} & \quad \text{Thread 2} \\
0: & \quad X := X+1 & 0: & \quad \text{IF IS_ODD}(X) \text{ THEN STOP_ALL} \\
1: & \quad \text{GOTO} \ 0 & 1: & \quad \text{GOTO} \ 0
\end{align*}
\]

The program is executed by repeatedly carrying out execution steps, where one thread is non-deterministically selected, its entire current line is run, and its program counter is then updated appropriately. This continues until \text{STOP_ALL} is executed, which immediately terminates the whole program.

(a) The program state can be described by \((pc_1, pc_2, X, stopped)\), where \(pc_1, pc_2, X\) are mathematical integers, and \(stopped\) is a boolean which is true iff \text{STOP_ALL} has been executed. Let \(S\) be the set of all such states.

(i) Define \(S_0\), the set of initial states of the program, such that \(S_0 \subseteq S\). [1 mark]

(ii) Define a transition relation \(R \subseteq S \times S\) describing the program’s execution. [2 marks]

(iii) Define a labelling function \(L\) that labels all states where the program has terminated with the atomic property \(\text{term}\). [2 marks]

(b) Explain why, taking the definitions from (a), the model \(M_a = (S, S_0, R, L)\) is not a (finite) Kripke structure. [2 marks]

(c) Draw the finite state automaton for a model \(M_b\) which is a Kripke structure, such that \(M_a\) and \(M_b\) are bisimilar. Justify your answer briefly. [5 marks]

[Note: A full formal proof of bisimilarity is not required.]

(d) (i) Give an LTL formula \(\phi\) such that the judgement \(M_b \models \phi\) corresponds to the statement “every execution of the program will eventually terminate”. [2 marks]

(ii) Either prove that \(M_b \models \phi\) holds, or describe a counter-example trace. [2 marks]

(e) Consider the CTL formula \(\psi = \text{AG}(\text{EF term})\). Determine whether this is equivalent to your definition of \(\phi\) from Part (d). \(\phi\) and \(\psi\) are equivalent iff, for all Kripke structures \(M\), \((M \models \phi)\) iff \((M \models \psi)\). Justify your answer. [4 marks]