This question is about modelling a program, defined below, consisting of two threads and a single (mathematical) integer variable \( X \), initially set to 0. Each thread \( t \) has its own program counter given by \( pc_t \), initially set to 0, which describes the current line for that thread.

\[
\begin{align*}
\text{Thread 1} & \quad \text{Thread 2} \\
0: & \quad X := X+1 & 0: & \quad \text{IF IS_ODD}(X) \text{ THEN STOP} \text{ ALL} \\
1: & \quad \text{GOTO} \ 0 & 1: & \quad \text{GOTO} \ 0
\end{align*}
\]

The program is executed by repeatedly carrying out execution steps, where one thread is non-deterministically selected, its entire current line is run, and its program counter is then updated appropriately. This continues until \text{STOP\_ALL} is executed, which immediately terminates the whole program.

(a) The program state can be described by \((pc_1, pc_2, X, stopped)\), where \( pc_1, pc_2, \) and \( X \) are mathematical integers, and \( stopped \) is a boolean which is true iff \text{STOP\_ALL} has been executed. Let \( S \) be the set of all such states.

\( \begin{align*}
(i) & \quad \text{Define } S_0, \text{ the set of initial states of the program, such that } S_0 \subseteq S. \ [1 \text{ mark}] \\
(ii) & \quad \text{Define a transition relation } R \subseteq S \times S \text{ describing the program’s execution. } \ [2 \text{ marks}] \\
(iii) & \quad \text{Define a labelling function } L \text{ that labels all states where the program has terminated with the atomic property } \text{term}. \ [2 \text{ marks}] \\
\end{align*} \)

(b) Explain why, taking the definitions from (a), the model \( M_a = (S, S_0, R, L) \) is \textit{not} a (finite) Kripke structure. \ [2 marks]

(c) Draw the finite state automaton for a model \( M_b \) which \textit{is} a Kripke structure, such that \( M_a \) and \( M_b \) are bisimilar. Justify your answer briefly. [Note: A full formal proof of bisimilarity is not required.] \ [5 marks]

(d) (i) Give an LTL formula \( \phi \) such that the judgement \( M_b \models \phi \) corresponds to the statement “every execution of the program will eventually terminate”. \ [2 marks]

(ii) Either prove that \( M_b \models \phi \) holds, or describe a counter-example trace. \ [2 marks]

(e) Consider the CTL formula \( \psi = \text{AG(EF term)} \). Determine whether this is equivalent to your definition of \( \phi \) from Part (d). \( \phi \) and \( \psi \) are equivalent iff, for all Kripke structures \( M \), \( (M \models \phi) \) iff \( (M \models \psi) \). Justify your answer. \ [4 marks]