7 Denotational Semantics (amp12)

(a) Suppose that \((D, \sqsubseteq)\) is a poset which is chain-complete but does not have a least element, and that \(f : D \to D\) is a continuous function.

(i) Give an example of such \((D, \sqsubseteq)\) and \(f\) for which \(f\) has no fixed point.  

[1 mark]

(ii) If \(d \in D\) satisfies \(d \sqsubseteq f(d)\), prove that there is a least element \(e \in D\) satisfying \(d \sqsubseteq e = f(e)\). [Hint: consider the method used to prove Tarski’s fixed point theorem.]  

[7 marks]

(b) (i) Define the notion of contextual equivalence for the language PCF. (You need not describe the syntax and semantics of PCF.)  

[2 marks]

(ii) State the compositionality, soundness and adequacy properties of the denotational semantics of PCF. Explain why they imply that any two closed PCF terms of the same type with equal denotations are contextually equivalent.  

[8 marks]

(iii) Give, without proof, an example of two contextually equivalent PCF terms that have unequal denotation.  

[2 marks]