COMPUTER SCIENCE TRIPOS Part II – 2019 – Paper 9

7 Denotational Semantics (amp12)

(*a*) Suppose that $(D, \sqsubseteq)$ is a poset which is chain-complete but does not have a least element, and that $f : D \to D$ is a continuous function.

(*i*) Give an example of such $(D, \sqsubseteq)$ and f for which f has no fixed point. [1 mark]

(*ii*) If $d \in D$ satisfies $d \sqsubseteq f(d)$, prove that there is a least element $e \in D$ satisfying $d \sqsubseteq e = f(e)$. [Hint: consider the method used to prove Tarski's fixed point theorem.] [7 marks]

(*b*) (*i*) Define the notion of *contextual equivalence* for the language PCF. (You need not describe the syntax and semantics of PCF.) [2 marks]

(*ii*) State the *compositionality*, *soundness* and *adequacy* properties of the denotational semantics of PCF. Explain why they imply that any two closed PCF terms of the same type with equal denotations are contextually equivalent. [8 marks]

(*iii*) Give, without proof, an example of two contextually equivalent PCF terms that have unequal denotation. [2 marks]