14  Types (nk480)

(a) In System F, give a Church encoding for (i) the Boolean type, (ii) the definition of the \texttt{True} and \texttt{False} constants, and (iii) the type and definition of the \texttt{if-then-else} operation. \hfill [3 marks]

(b) In System F, give (i) a Church encoding \texttt{Nat} for the natural numbers, (ii) a Church encoding for the \texttt{Zero} : \texttt{Nat} and \texttt{Succ} : \texttt{Nat} \rightarrow \texttt{Nat} constructors, and (iii) a type and definition for the iteration operator \texttt{Iter} for natural numbers. \hfill [3 marks]

(c) (i) In System F, give a Church encoding for (i) an \texttt{Option}_A type, (ii) the definitions of the \texttt{None} : \texttt{Option}_A and \texttt{Some} : \texttt{A} \rightarrow \texttt{Option}_A operations, and (iii) the type and definition of the \texttt{case} operation on options.

(ii) Assume that \texttt{n} : \texttt{B} and \texttt{s} : \texttt{A} \rightarrow \texttt{B}, and then

(A) Prove that \texttt{Case[B] n s None} = \texttt{n}

(B) Prove that \texttt{Case[B] n s (Some x)} = \texttt{s x} \hfill [5 marks]

(d) In System F, define a predecessor operation \texttt{Pred} : \texttt{Nat} \rightarrow \texttt{Nat}, which returns \texttt{Zero} if given \texttt{Zero} as an argument, and return \texttt{n} if given \texttt{Succ n} as an argument. \hfill [Hint: The option type may be useful in formulating this definition.] \hfill [8 marks]

(e) In System F, define a subtraction operator \texttt{Sub} : \texttt{Nat} \rightarrow \texttt{Nat} \rightarrow \texttt{Nat}, which is defined to be \textit{saturating}. That is, \texttt{Sub m n} returns the difference if \texttt{m} \geq \texttt{n}, and returns 0 otherwise. \hfill [1 mark]