14 Types (nk480)

(a) In System F, give a Church encoding for (i) the Boolean type, (ii) the definition of the True and False constants, and (iii) the type and definition of the if-then-else operation. [3 marks]

(b) In System F, give (i) a Church encoding Nat for the natural numbers, (ii) a Church encoding for the Zero : Nat and Succ : Nat → Nat constructors, and (iii) a type and definition for the iteration operator Iter for natural numbers. [3 marks]

(c) (i) In System F, give a Church encoding for (i) an OptionA type, (ii) the definitions of the None : OptionA and Some : A → OptionA operations, and (iii) the type and definition of the case operation on options.

   (ii) Assume that n : B and s : A → B, and then

       (A) Prove that Case [B] n s None = n

       (B) Prove that Case [B] n s (Some x) = s x

   [5 marks]

(d) In System F, define a predecessor operation Pred : Nat → Nat, which returns Zero if given Zero as an argument, and return n if given Succ n as an argument. [Hint: The option type may be useful in formulating this definition.] [8 marks]

(e) In System F, define a subtraction operator Sub : Nat → Nat → Nat, which is defined to be saturating. That is, Sub m n returns the difference if \( m \geq n \), and returns 0 otherwise. [1 mark]