COMPUTER SCIENCE TRIPOS Part II – 2019 – Paper 9

13 Quantum Computing (ad260)

A Boolean formula ϕ with n variables in it can be seen as defining a function $f : \{0,1\}^n \to \{0,1\}$, and we say that ϕ is satisfiable if there is some $x \in \{0,1\}^n$ such that $f(x) = 1$.

(*a*) Explain how f can be suitably represented as a unitary operation U_f on a complex space of dimension 2^{n+1}. [3 marks]

(*b*) Suppose that we are given a blackbox implementing U_f. Describe how this would be used to form the *Grover iterate* which can be repeated to find a value x such that $f(x) = 1$. [5 marks]

(*c*) If there is exactly one value x such that $f(x) = 1$, how many iterations of the Grover iterate would you use to find this value? What is the probability of finding it? [3 marks]

(*d*) If there are M distinct values such that $f(x) = 1$, how many iterations of the Grover iterate would you use to find one of these values? What is the probability of finding one of them? [3 marks]

(*e*) If you are able to turn an arbitrary formula ϕ into an implementation of the corresponding unitary operator U_f, how would you use this to give an algorithm for determining whether ϕ is satisfiable or not? Give an estimate of the running time of your algorithm in terms of n. [6 marks]