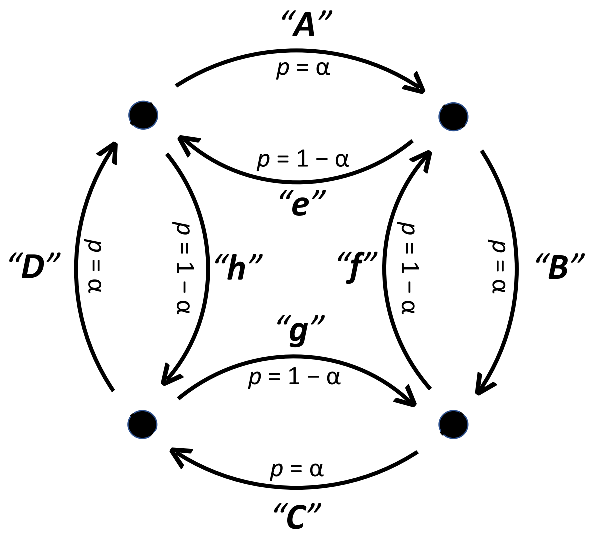
8 Information Theory (jgd1000)

(*a*) Consider the four-state Markov process in the graph below. It emits the eight letters $\{A, B, C, D, e, f, g, h\}$ with probabilities and changes of state as shown, but note the sequence constraints. (For example, an A can only be followed by a B or an e.) Letter emissions with clockwise state transitions occur with probability α, and the others with probability $1 - \alpha$, where $0 < \alpha < 1$.



(*i*) First imagine a one-state Markov process that emits any of eight letters with equal probabilities. What is its entropy? [2 marks]

(*ii*) For the four-state Markov process shown with parameter α, what is the long-term probability distribution across the eight letters? [4 marks]

(*iii*) In terms of parameter α, what is the overall entropy $H(\alpha)$ of this four-state Markov process? [2 marks]

(*iv*) Sketch a plot of $H(\alpha)$ as a function of α. Compare its maximum value with your earlier answer in Part (*a*)(*i*) for a one-state Markov process that also emits eight letters, and explain the difference, if any. [4 marks]

(*b*) Is it possible to construct an instantaneous code (a code possessing the prefix property) for a five-letter symbol set using codewords whose lengths in bits are: 1, 2, 3, 3, and 4 bits? Justify your answer by stating the relevant condition. [4 marks]

(*c*) Provide an operation in linear algebra that involves simply the multiplication of a matrix by a vector, which describes the Discrete Fourier Transform of a discrete sequence of data $f[n] = (f[1], ..., f[N])$ to obtain Fourier coefficients $F[k] = (F[1], ..., F[N])$. Define the elements of the $(N \times N)$ matrix and give the computational cost of the operation in this vector-matrix form. [4 marks]