(a) Consider the four-state Markov process in the graph below. It emits the eight letters \{A, B, C, D, e, f, g, h\} with probabilities and changes of state as shown, but note the sequence constraints. (For example, an A can only be followed by a B or an e.) Letter emissions with clockwise state transitions occur with probability \(\alpha\), and the others with probability \(1 - \alpha\), where \(0 < \alpha < 1\).

(i) First imagine a one-state Markov process that emits any of eight letters with equal probabilities. What is its entropy? \[2\text{ marks}\]

(ii) For the four-state Markov process shown with parameter \(\alpha\), what is the long-term probability distribution across the eight letters? \[4\text{ marks}\]

(iii) In terms of parameter \(\alpha\), what is the overall entropy \(H(\alpha)\) of this four-state Markov process? \[2\text{ marks}\]

(iv) Sketch a plot of \(H(\alpha)\) as a function of \(\alpha\). Compare its maximum value with your earlier answer in Part (a)(i) for a one-state Markov process that also emits eight letters, and explain the difference, if any. \[4\text{ marks}\]

(b) Is it possible to construct an instantaneous code (a code possessing the prefix property) for a five-letter symbol set using codewords whose lengths in bits are: 1, 2, 3, 3, and 4 bits? Justify your answer by stating the relevant condition. \[4\text{ marks}\]

(c) Provide an operation in linear algebra that involves simply the multiplication of a matrix by a vector, which describes the Discrete Fourier Transform of a discrete sequence of data \(f[n] = (f[1], ..., f[N])\) to obtain Fourier coefficients \(F[k] = (F[1], ..., F[N])\). Define the elements of the \((N \times N)\) matrix and give the computational cost of the operation in this vector-matrix form. \[4\text{ marks}\]