Recall the three judgements for classical propositional logic:

(a) \( \Gamma ; \Delta \vdash e : A \text{ true} \) – \( e \) is a proof of type \( A \)

(b) \( \Gamma ; \Delta \vdash k : A \text{ false} \) – \( k \) is a refutation of type \( A \)

(c) \( \Gamma ; \Delta \vdash \langle e \mid_A k \rangle \text{ contr} \) – \( \langle e \mid_A k \rangle \) is a contradiction at type \( A \)

Here, \( \Gamma \) contains the true assumptions, and \( \Delta \) are the false assumptions. In this question, we will extend classical propositional logic with support for the implication or function type operator \( A \rightarrow B \).

(a) Give a proof term and inference rule for a proof of type \( A \rightarrow B \). [4 marks]

(b) Give a proof term and inference rule for a refutation of type \( A \rightarrow B \).

[HINT: how is implication encoded in classical logic?] [4 marks]

(c) Give a reduction rule for contradiction configurations of the form \( \langle e \mid_{A \rightarrow B} k \rangle \). [4 marks]

(d) (i) State the preservation theorem for classical logic. [2 marks]

(ii) Give the proof of preservation for the case of the new rule defined above. You may assume that weakening, exchange and substitution all hold. [6 marks]