5 Computation Theory (amp12)

For each $e \in \mathbb{N}$, let φ_e denote the partial function $\mathbb{N} \rightharpoonup \mathbb{N}$ computed by the register machine with index e.

(a) What is meant by a *universal register machine* for computing partial functions $\mathbb{N}^k \rightharpoonup \mathbb{N}$ of any number of arguments k. [3 marks]

(b) How would you modify the machine from Part (a) to compute the partial function $u : \mathbb{N}^2 \rightharpoonup \mathbb{N}$ satisfying $u(e, x) \equiv \varphi_e(x)$ for all $e, x \in \mathbb{N}$? [2 marks]

(c) Given a register machine computable partial function $g : \mathbb{N}^2 \rightharpoonup \mathbb{N}$, show that there is a *total* function $\bar{g} : \mathbb{N} \to \mathbb{N}$ which is register machine computable and which satisfies $u(\bar{g}(x), y) \equiv g(x, y)$ for all $x, y \in \mathbb{N}$. [7 marks]

(d) Suppose $h : \mathbb{N} \to \mathbb{N}$ is a total function which is register machine computable. Show that there exists a number $n \in \mathbb{N}$ such that φ_n and $\varphi_{h(n)}$ are equal partial functions.
[*Hint*: let g be the computable partial function defined by $g(x, y) \equiv u(h(u(x, x)), y)$ and consider $\bar{g}(e)$ where $\bar{g}$ is the function obtained from g as in Part (c) and e is the index of some register machine that computes it.]
[8 marks]