For each \( e \in \mathbb{N} \), let \( \varphi_e \) denote the partial function \( \mathbb{N} \rightarrow \mathbb{N} \) computed by the register machine with index \( e \).

(a) What is meant by a universal register machine for computing partial functions \( \mathbb{N}^k \rightarrow \mathbb{N} \) of any number of arguments \( k \). [3 marks]

(b) How would you modify the machine from Part (a) to compute the partial function \( u : \mathbb{N}^2 \rightarrow \mathbb{N} \) satisfying \( u(e, x) \equiv \varphi_e(x) \) for all \( e, x \in \mathbb{N} \)? [2 marks]

(c) Given a register machine computable partial function \( g : \mathbb{N}^2 \rightarrow \mathbb{N} \), show that there is a total function \( \bar{g} : \mathbb{N} \rightarrow \mathbb{N} \) which is register machine computable and which satisfies \( u(\bar{g}(x), y) \equiv g(x, y) \) for all \( x, y \in \mathbb{N} \). [7 marks]

(d) Suppose \( h : \mathbb{N} \rightarrow \mathbb{N} \) is a total function which is register machine computable.
   Show that there exists a number \( n \in \mathbb{N} \) such that \( \varphi_n \) and \( \varphi_{h(n)} \) are equal partial functions.
   [Hint: let \( g \) be the computable partial function defined by \( g(x, y) \equiv u(h(u(x, x)), y) \) and consider \( \bar{g}(e) \) where \( \bar{g} \) is the function obtained from \( g \) as in Part (c) and \( e \) is the index of some register machine that computes it.] [8 marks]