4 Complexity Theory (ad260)

A Boolean formula ϕ is in *conjunctive normal form* (CNF) if it is the conjunction of clauses, each of which is the disjunction of literals. It is said to be in k-CNF (for $k \in \mathbb{N}$) if each clause has exactly k literals in it.

An assignment $\sigma : V \to \{\text{true}, \text{false}\}$ of truth values to the variables is a *satisfying assignment* for a CNF formula ϕ if it makes at least one literal in each clause of ϕ true. It is said to be a *not-all-equals* assignment for ϕ if it makes at least one literal in each clause of ϕ true *and* at least one literal in each clause of ϕ false.

Let CNF-SAT denote the problem of determining, given a formula in CNF, whether it has a satisfying assignment.

Let k-SAT denote the problem of determining, given a formula in k-CNF, whether it has a satisfying assignment.

Let k-NAE denote the problem of determining, given a formula in k-CNF, whether it has a not-all-equals assignment.

(*a*) Explain why CNF-SAT is NP-complete. Your explanation should include a full definition of NP-completeness and a brief sketch of the proof of the Cook-Levin theorem. [5 marks]

(*b*) Show that 3-SAT is NP-complete by means of a suitable reduction. [3 marks]

(*c*) Give a polynomial-time reduction from 3-SAT to 4-NAE. What can you conclude about the complexity of the latter problem?
(*Hint:* consider introducing one new variable and adding it to every clause.) [8 marks]

(*d*) Show that the problem 3-NAE is NP-complete.
(*Hint:* consider a reduction from 4-NAE) [4 marks]