A Boolean formula \( \phi \) is said to be *satisfiable* if there is an assignment \( \sigma : V \rightarrow \{\text{true}, \text{false}\} \) of values to the variables of \( \phi \) that makes it true.

A *quantified Boolean formula* \( \theta \) is an expression that is (i) either a Boolean formula; or (ii) \( \exists X \phi \) where \( \phi \) is a quantified Boolean formula and \( X \) is variable; or (iii) \( \forall X \phi \) where \( \phi \) is a quantified Boolean formula and \( X \) is variable.

We say that a quantified Boolean formula \( \theta \) is satisfied by an assignment \( \sigma : V \rightarrow \{\text{true}, \text{false}\} \) if either

- \( \theta \) is a Boolean formula that is made true by \( \sigma \); or
- \( \theta \) is \( \exists X \phi \) and either \( \sigma[X/\text{true}] \) or \( \sigma[X/\text{false}] \) make \( \phi \) true; or
- \( \theta \) is \( \forall X \phi \) and both \( \sigma[X/\text{true}] \) and \( \sigma[X/\text{false}] \) make \( \phi \) true.

Here, \( \sigma[X/v] \) denotes the assignment that is the same as \( \sigma \) for all variables apart from \( X \), and it maps \( X \) to the truth value \( v \).

We write QBF for the decision problem of determining whether a given quantified Boolean formula is satisfiable. In answering the questions below, you may assume the NP-completeness of any standard problem, as long as you state your assumptions clearly.

(a) Show that QBF is NP-hard. [4 marks]
(b) Show that QBF is co-NP-hard. [6 marks]
(c) Show that QBF is in PSPACE. [6 marks]
(d) Is QBF NP-complete? Why or why not? [4 marks]