10 Logic and Proof (lp15)

(a) For each of the following formulas, present either a formal resolution proof or a falsifying interpretation. Note that $a$ and $b$ are constants.

\[
\forall x [Q(x) \to R(x)] \land \neg R(a) \land \forall x [\neg R(x) \land \neg Q(x) \to P(b) \lor Q(b)] \to P(b) \lor R(b)
\]

[4 marks]

\[
\exists x [\forall y z [(P(y) \to Q(z)) \to (P(x) \to Q(x))]]
\]

[4 marks]

(b) For each of the following formulas, present a proof in a sequent or tableau calculus, or alternatively, a falsifying interpretation. In Part (b)(iii) the modal logic is S4.

(i) $\exists y \forall x P(x, y) \to \exists z P(z, z)$  

[3 marks]

(ii) $\forall x [P(x) \land \exists y \neg P(y)] \to Q$  

[5 marks]

(iii) $(\Box \Diamond P \land \Box \Diamond Q) \to \Box \Diamond (P \land Q)$  

[4 marks]