3 Introduction to Graphics (rkm38)

A homographic transformation of a point in a 3D space is expressed as:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

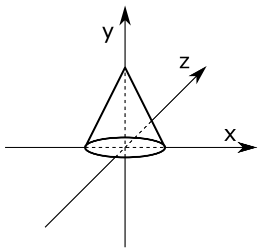
(*a*) For each of the following matrices, identify a basic transformation (translation, scaling, rotation, projection) or a sequence of them that a point will undergo when multiplied by that matrix. Name any axis of rotation and list the transformations in the correct order.

$$(i) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (ii) \begin{bmatrix} \cos\theta & 0 & \sin\theta & 3 \\ 0 & 1 & 0 & 7 \\ -\sin\theta & 0 & \cos\theta & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 2\cos\theta & -\sin\theta & 0 & 0 \\ 2\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (iv) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

[8 marks]

(*b*) You have a cone of height one unit; apex at $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$; and the base of diameter one unit centred at the origin. This cone is shown in the figure below.



You wish to transform this cone so that the apex is at $\begin{pmatrix} -1 & 2 & -3 \end{pmatrix}$, the base is centred at $\begin{pmatrix} 1 & 4 & -3 \end{pmatrix}$, and the base's diameter is two units. What transformations are required to achieve this and in what order should they be performed? Write down the product of individual transformation matrices. There is no need to multiply them together to compute the combined transformation. [12 marks]