Consider formal languages $L(a) \subset \Sigma^*$ over the alphabet $\Sigma = \{0, 1\}$.

(a) $L(a) \subset \Sigma^*$ consists of all and only the strings with an even number of 0s.

(i) Build a regular expression for $L(a)$. [2 marks]

(ii) Draw the transition graph of a deterministic finite automaton (DFA) for $L(a)$. [2 marks]

(b) $L(b) \subset \Sigma^*$ is defined by the following axiom and rules:

$$\begin{align*}
0 & \rightarrow u \\
001 & \rightarrow u \\
w01 & \rightarrow wu \\
w10 & \rightarrow w1u
\end{align*}$$

where $u$ and $w$ are string variables in $\Sigma^*$ while 0 and 1 are literals.

(i) State a property $P_1$ enjoyed by all strings in $L(b)$ but by none of the following strings: 01011, 1, 111, 111110001100, 10101. [2 marks]

(ii) Prove that all strings in $L(b)$ enjoy the property $P_1$ you defined in your answer to Part (b)(i). [3 marks]

(iii) Either prove the following statement or provide a counterexample: “There is no string in $L(b)$ with two consecutive 1s”. [4 marks]

(c) Language $L(c) \subset \Sigma^*$ consists of the strings that enjoy the following four properties simultaneously:

- $P_2$ : “having a number of 0s divisible by three”;
- $P_3$ : “including the 11011 substring”;
- $P_4$ : “having at least four 0s”;
- $P_5$ : “having no more than five 1s”.

(i) Give three minimum-length strings in $L(c)$. [1 mark]

(ii) For each of the properties $P_2$–$P_5$, draw the transition diagram for a matching DFA. [4 marks]

(iii) Describe how to build a DFA for $L(c)$ by combining the ones you built for Part (c)(ii). [2 marks]