10   Discrete Mathematics (fms27)

Consider formal languages $L_{(x)}$ over the alphabet $\Sigma = \{0, 1\}$.

(a) $L_{(a)} \subset \Sigma^*$ consists of all and only the strings with an even number of 0s.

   (i)   Build a regular expression for $L_{(a)}$.   [2 marks]

   (ii) Draw the transition graph of a deterministic finite automaton (DFA) for $L_{(a)}$.   [2 marks]

(b) $L_{(b)} \subset \Sigma^*$ is defined by the following axiom and rules:

$$\frac{}{0} \qquad \frac{u}{001u} \qquad \frac{w10u}{wu}$$

where u and w are string variables in Σ^* while 0 and 1 are literals.

   (i)   State a property P_1 enjoyed by all strings in $L_{(b)}$ but by none of the following strings: 01011, 1, 111, 11111100001100, 10101.   [2 marks]

   (ii) Prove that all strings in $L_{(b)}$ enjoy the property P_1 you defined in your answer to Part $(b)(i)$.   [3 marks]

   (iii) Either prove the following statement or provide a counterexample: "There is no string in $L_{(b)}$ with two consecutive 1s".   [4 marks]

(c) Language $L_{(c)} \subset \Sigma^*$ consists of the strings that enjoy the following four properties simultaneously:

   • P_2 : "having a number of 0s divisible by three";

   • P_3 : "including the 11011 substring";

   • P_4 : "having at least four 0s";

   • P_5 : "having no more than five 1s".

   (i)   Give three minimum-length strings in $L_{(c)}$.   [1 mark]

   (ii) For each of the properties P_2–P_5, draw the transition diagram for a matching DFA.   [4 marks]

   (iii) Describe how to build a DFA for $L_{(c)}$ by combining the ones you built for Part $(c)(ii)$.   [2 marks]