Consider formal languages \( L_x \) over the alphabet \( \Sigma = \{0, 1\} \).

(a) \( L_a \subset \Sigma^* \) consists of all and only the strings with an even number of 0s.

(i) Build a regular expression for \( L_a \). [2 marks]

(ii) Draw the transition graph of a deterministic finite automaton (DFA) for \( L_a \). [2 marks]

(b) \( L_b \subset \Sigma^* \) is defined by the following axiom and rules:

\[
0 \quad u \quad w10u \quad wu
\]

where \( u \) and \( w \) are string variables in \( \Sigma^* \) while 0 and 1 are literals.

(i) State a property \( P_1 \) enjoyed by all strings in \( L_b \) but by none of the following strings: 01011, 1, 111, 11111100001100, 10101. [2 marks]

(ii) Prove that all strings in \( L_b \) enjoy the property \( P_1 \) you defined in your answer to Part (b)(i). [3 marks]

(iii) Either prove the following statement or provide a counterexample: “There is no string in \( L_b \) with two consecutive 1s”. [4 marks]

(c) Language \( L_c \subset \Sigma^* \) consists of the strings that enjoy the following four properties simultaneously:

- \( P_2 \) : “having a number of 0s divisible by three”;
- \( P_3 \) : “including the 11011 substring”;
- \( P_4 \) : “having at least four 0s”;
- \( P_5 \) : “having no more than five 1s”.

(i) Give three minimum-length strings in \( L_c \). [1 mark]

(ii) For each of the properties \( P_2 \)–\( P_5 \), draw the transition diagram for a matching DFA. [4 marks]

(iii) Describe how to build a DFA for \( L_c \) by combining the ones you built for Part (c)(ii). [2 marks]