10 Discrete Mathematics (fms27)

Consider formal languages $L_{(x)}$ over the alphabet $\Sigma = \{0, 1\}$.

(*a*) $L_{(a)} \subset \Sigma^*$ consists of all and only the strings with an even number of 0s.

(*i*) Build a regular expression for $L_{(a)}$. [2 marks]

(*ii*) Draw the transition graph of a deterministic finite automaton (DFA) for $L_{(a)}$. [2 marks]

(*b*) $L_{(b)} \subset \Sigma^*$ is defined by the following axiom and rules:

$$\frac{}{0} \qquad \frac{u}{001u} \qquad \frac{w10u}{wu}$$

where u and w are string variables in Σ^* while 0 and 1 are literals.

(*i*) State a property P_1 enjoyed by all strings in $L_{(b)}$ but by none of the following strings: 01011, 1, 111, 11111100001100, 10101. [2 marks]

(*ii*) Prove that all strings in $L_{(b)}$ enjoy the property P_1 you defined in your answer to Part (*b*)(*i*). [3 marks]

(*iii*) Either prove the following statement or provide a counterexample: "There is no string in $L_{(b)}$ with two consecutive 1s". [4 marks]

(*c*) Language $L_{(c)} \subset \Sigma^*$ consists of the strings that enjoy the following four properties simultaneously:

- P_2 : "having a number of 0s divisible by three";
- P_3 : "including the 11011 substring";
- P_4 : "having at least four 0s";
- P_5 : "having no more than five 1s".

(*i*) Give three minimum-length strings in $L_{(c)}$. [1 mark]

(*ii*) For each of the properties P_2–P_5, draw the transition diagram for a matching DFA. [4 marks]

(*iii*) Describe how to build a DFA for $L_{(c)}$ by combining the ones you built for Part (*c*)(*ii*). [2 marks]