COMPUTER SCIENCE TRIPOS Part IA – 2019 – Paper 1

9 Algorithms (djw1005)

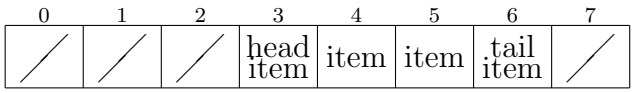
A Random Access Queue supports the operations `pushright(`x`)` to add a new item x to the tail, `popleft()` to remove the item at the head, and `element_at(`i`)` to retrieve the item at position i without removing it: $i = 0$ gives the item at the head, $i = 1$ the following element, and so on.

(a) We can implement this data structure using a simple linked list, where `element_at(`i`)` iterates from the head of the list until it reaches position i.

(i) State the complexity of each of the three operations. [1 mark]

(ii) A colleague suggests that, by defining a clever potential function, it might be possible to show that all operations have amortized cost $O(1)$. Show carefully that your colleague is mistaken. [6 marks]

(b) We can also implement this data structure using an array. The picture below shows a queue holding 4 items, stored within an array of size 8. When new items are pushed, it may be necessary to create a new array and copy the queue into it. The cost of creating an array of size n is $\Theta(n)$.



(i) Give pseudocode for the three operations. Each operation should have amortized cost $O(1)$. [6 marks]

(ii) Prove that the amortized costs of your operations are indeed $O(1)$. [7 marks]