9 Algorithms (djw1005)

A Random Access Queue supports the operations pushright($x$) to add a new item $x$ to the tail, popleft() to remove the item at the head, and element_at($i$) to retrieve the item at position $i$ without removing it: $i = 0$ gives the item at the head, $i = 1$ the following element, and so on.

(a) We can implement this data structure using a simple linked list, where element_at($i$) iterates from the head of the list until it reaches position $i$.

(i) State the complexity of each of the three operations. [1 mark]

(ii) A colleague suggests that, by defining a clever potential function, it might be possible to show that all operations have amortized cost $O(1)$. Show carefully that your colleague is mistaken. [6 marks]

(b) We can also implement this data structure using an array. The picture below shows a queue holding 4 items, stored within an array of size 8. When new items are pushed, it may be necessary to create a new array and copy the queue into it. The cost of creating an array of size $n$ is $\Theta(n)$.

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\hline & & & \text{head} & \text{item} & \text{item} & \text{item} & \text{tail} & \hline
\end{array}
\]

(i) Give pseudocode for the three operations. Each operation should have amortized cost $O(1)$. [6 marks]

(ii) Prove that the amortized costs of your operations are indeed $O(1)$. [7 marks]