COMPUTER SCIENCE TRIPOS Part IA – 2019 – Paper 1

1 Foundations of Computer Science (am21)

Three alternative representations for non-negative integers, n, are:

- **Peano**: values have the form `S(... S(Z) ...)`, applying `S` n times to `Z` where `S` and `Z` are constructors or constants of some data type.
- **Binary**: values are of type `bool list` with 0 being represented as the empty list, and the least-significant bit being stored in the head of the list.
- **Church**: values have the form `fn f => fn x => f(... f(x) ...)`, applying `f` n times to `x`

(a) Write ML functions for *each* of these data types which take the representation of an integer n as argument and return n as an ML `int`. [6 marks]

(b) Write ML functions for *each* of these data types which take representations of integers m and n and return the representation of $m + n$. Your answers must *not* use any value or operation on type `int` or `real`. [*Hint:* you might it useful to write a function `majority: bool*bool*bool -> bool` (which returns true when two or more of its arguments are true) and to note that the ML inequality operator '`<>`' acts as exclusive-or on `bool`.] [10 marks]

(c) Letting *two* and *three* respectively be the Church representations of integers 2 and 3, indicate whether each of the following ML expressions give a Church representation of some integer and, if so what integer is represented, and if not giving a one-line reason.

(i) *two three*

(ii) *three two*

(iii) *two* $\circ$ *three*

(iv) *three* $\circ$ *two*

[4 marks]