6 Digital Signal Processing (MGK)

(a) When converting a digital audio signal from one sampling frequency to another, it is common practice to use a low-pass filter. What is the purpose of this low-pass filter, and what cut-off frequency should it have if the change of sampling frequency is

(i) from 12 kHz to 48 kHz;

(ii) from 48 kHz to 12 kHz. [4 marks]

(b) You are working on the firmware of a quadcopter drone. Your colleague, through trial and error, found that the following recursive filter nicely avoids unwanted oscillations in the control system:

\[
\begin{align*}
\hat{z}^{-1} & \quad e & \quad d & \quad e \\
\quad & \quad v_n & \quad f & \quad y_n \\
x_n & \quad & \quad & \\
\end{align*}
\]

(i) What are the first three samples \(h_0, h_1, h_2\) of the impulse response of this filter? [Note: All delay elements have been initialized to zero.] [6 marks]

(ii) What is the \(z\)-transform \(H(z) = Y(z)/X(z)\) of the impulse response of this digital filter? [5 marks]

(iii) The software development kit of your flight controller can only implement digital filters of the form

\[
y_n = \sum_{k=0}^{3} b_k \cdot x_{n-k} - \sum_{l=1}^{3} a_l \cdot y_{n-l}.
\]

What coefficient values \(a_l\) and \(b_k\) (\(0 \leq k \leq 3, 1 \leq l \leq 3\)) will implement the same impulse response as your colleague’s filter? [5 marks]