COMPUTER SCIENCE TRIPOS Part II – 2018 – Paper 9

5 Denotational Semantics (MPF)

- (a) Give the definitions of poset (partially ordered set), cpo (chain complete poset), and domain. [2 marks]
- (b) An *ideal* I of a poset $P = (P, \leq)$ is a subset I of the set P such that:
 - *I* is non-empty,
 - for every $x \in I$ and $y \in P$, $y \leq x$ implies $y \in I$, and
 - for every $x, y \in I$, there is $u \in I$ such that $x \leq u$ and $y \leq u$.

We write Idl(P), where P is a poset, for the set of all the ideals of P.

- (i) Show that $(Idl(\mathsf{P}), \subseteq)$, where \subseteq denotes the subset-inclusion relation, is a cpo. [3 marks]
- (*ii*) Show that $Idl(P, \leq)$ is non-empty iff P is non-empty. [2 marks]
- (*iii*) Give a necessary and sufficient condition on a poset P for the cpo $(Idl(\mathsf{P}), \subseteq)$ to be a domain. Justify your answer. [2 marks]
- (c) Give the definitions of monotone function between posets and of continuous function between cpos. [2 marks]
- (d) (i) For a monotone function $f : \mathsf{P} \to \mathsf{Q}$ between posets P and Q , define a continuous function $f^{\#} : (Idl(\mathsf{P}), \subseteq) \to (Idl(\mathsf{Q}), \subseteq)$ between the cpos $(Idl(\mathsf{P}), \subseteq)$ and $(Idl(\mathsf{Q}), \subseteq)$. Prove that your definition is as requested. [4 marks]
 - (*ii*) For the identity function id_{P} on a poset P , show that $(id_{\mathsf{P}})^{\#}$ is the identity function on the cpo $(Idl(\mathsf{P}), \subseteq)$. [1 mark]
 - (*iii*) For monotone functions $f : \mathsf{P} \to \mathsf{Q}$ and $g : \mathsf{O} \to \mathsf{P}$ between posets O, P , and Q , show that $f^{\#} \circ g^{\#} = (f \circ g)^{\#} : (Idl(\mathsf{O}), \subseteq) \to (Idl(\mathsf{Q}), \subseteq).$ [4 marks]