

3 Computer Systems Modelling (RJG)

(a) Consider a birth death process $X(t)$ with state space $0, 1, 2, \dots$ and state dependent birth and death rates given by λ_i ($i \geq 0$) and μ_i ($i > 0$), respectively.

(i) Draw a state space diagram for the birth death process labelled with the transition rates. [2 marks]

(ii) Derive the Chapman-Kolmogorov equations for $P_i(t)$ ($i = 0, 1, \dots$) where

$$P_i(t) = \mathbb{P}(X(t) = i).$$

[4 marks]

(iii) Use the detailed balance method to derive the stationary distribution $p_i = \lim_{t \rightarrow \infty} P_i(t)$ for $i = 0, 1, \dots$. You should state any conditions required for the existence of the stationary distribution. [4 marks]

(b) Consider a data centre comprising N nodes forming a computer cluster. Suppose that the individual nodes are unreliable where the time a node runs before breaking down and needing repair is exponential with rate λ independent of other nodes. A single repairer is able to return a broken-down node to service in a time that is exponentially distributed with rate μ independent of other nodes. Model the number of broken down nodes by a birth death process, draw the state space diagram, state the birth and death rates of the model and determine its stationary distribution. Comment on whether the stationary distribution always exists. Given the stationary distribution derive an expression for the mean number of broken down nodes and the stationary long run probability that a given node is working. [10 marks]