3 Computer Systems Modelling (RJG)

This question deals with stochastic processes $\{N(t), t \geq 0\}$ where $N(t)$ represents the number of events in the time interval $[0, t]$.

(*a*) (*i*) Define a Poisson process $\{N(t), t \geq 0\}$ of rate $\lambda > 0$. [2 marks]

(*ii*) Show that $N(t) \sim \text{Pois}(\lambda t)$ for each fixed $t > 0$. You may use the result that $\lim_{n\to\infty}(1 - x/n)^n = e^{-x}$ without proof. [4 marks]

(*iii*) Let X_1 be the time of the first event of the Poisson process $N(t)$. Show that $X_1 \sim \text{Exp}(\lambda)$. [2 marks]

(*iv*) Now given that $N(t) = 1$ derive the distribution of the time of the single event in $[0, t]$. [4 marks]

(*b*) Suppose that events of a Poisson process of rate λ are independently selected at random with probability $p > 0$. Show that the process of selected events is also a Poisson process and establish its rate. [2 marks]

(*c*) Describe how your result from part (*b*) can be used to simulate a non-homogeneous Poisson process whose rate function $\lambda(t)$ is such that $\lambda(t) \leq \lambda^*$ for all $t \geq 0$. [6 marks]