11 Quantum Computing (AD)

(*a*) In the quantum teleportation protocol, Alice and Bob are each in possession of one qubit of a pair in the joint state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. In addition, Alice has a qubit in an arbitrary state $|\phi\rangle$. Explain how the protocol works. In particular, show that it involves the transmission of exactly two classical bits of information from Alice to Bob and demonstrate how, at the end of the protocol, Bob is in possession of a qubit in state $|\phi\rangle$. [10 marks]

(*b*) Suppose now that Alice has two qubits in a state

$$|\theta\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle.$$

In addition, there is a pair of qubits in the state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ such that Alice is in possession of one qubit of the pair while Bob is in possession of the other.

Alice now uses the quantum teleportation protocol to transmit to Bob the first qubit of $|\theta\rangle$. What is the resulting joint state of the two-qubit system composed of the second of Alice's qubits and the qubit in Bob's possession? [10 marks]