

6 Computation Theory (AD)

(a) What does it mean to say that a partial function  $f : \mathbb{N}^k \rightarrow \mathbb{N}$  is *register machine computable*? [2 marks]

(b) Show that the following functions are register machine computable:

(i)  $\text{add}(x, y) \triangleq x + y$ ;

(ii)  $\text{max}(x, y) \triangleq \begin{cases} y & \text{if } x \leq y \\ x & \text{otherwise} \end{cases}$  ; and

(iii)  $\text{comp}(x, y) \triangleq \begin{cases} 0 & \text{if } x \leq y \\ 1 & \text{otherwise.} \end{cases}$

[9 marks]

(c) What does it mean to say that a function  $f : \mathbb{N}^k \rightarrow \mathbb{N}$  is  *$\lambda$ -definable*? [2 marks]

(d) Is every  $\lambda$ -definable function register-machine computable? Give a detailed justification for your answer. [7 marks]